

Operations

Whole Numbers

Multiplication and Division

Operations Objectives (Draft)

Whole Numbers

SOL Link

Multiplication and Division:

Obj. 1	Use models to explore multiplication and division using addition of equal groups. Recognize and use language and symbols to represent multiplication and division.	CE 3.9
Obj. 2	Use manipulatives to explore multiplication using arrays. Recognize prime, composite, and square numbers.	CE 6.3
Obj. 3	Mentally compute multiplication facts through nines. Recognize the use of properties and strategies: commutative, zero, identity, doubles, fives, skip counting, and nines.	CE 3.9
Obj. 4	Multiply three single digit factors. Recognize and use the distributive and associative properties.	
Obj. 5	Use models to multiply 2-digit numbers by 1-digit numbers.	CE 3.10
Obj. 6	Multiply 1- and 2-digit numbers by powers and multiples of ten. Use sampling to estimate large quantities.	CE 4.8
Obj. 7	Estimate, model, and multiply 2-digit numbers by 2-digit numbers.	CE 4.8
Obj. 8	Estimate and multiply 3-digit numbers by 2-digit numbers.	CE 4.8
Obj. 9	Multiply whole numbers with 3 digits in each factor.	
Obj. 10	Write and evaluate numbers in exponential form.	PFA 6.22
Obj. 11	Use calculators to solve problems multiplying whole numbers.	CE 5.3

Obj. 12	Model and write division situations (1- and 2-digit numbers by 1- digit numbers) with and without remainders.	CE 4.9
Obj. 13	Mentally divide by 1-digit numbers and multiples of ten. Recognize patterns in division	
Obj. 14	Estimate and divide 2- and 3-digit numbers by 1-digit numbers with and without remainders. Use multiplication to check division.	CE 5.5
Obj. 15	Estimate and divide by 2-digit divisors.	CE 5.5
Obj. 16	Interpret the quotient and remainder.	CE 5.3
Obj. 17	Evaluate numerical expressions using order of operations.	
Obj. 18	Choose the operation to solve and create problems involving addition, subtraction, multiplication, and division of whole numbers.	CE 5.3

Objective 1: Use models to explore multiplication and division using addition of equal groups and arrays. Recognize and use language and symbols to represent multiplication and division.

Vocabulary

multi-
multiply
multiple
multiplication
divide
division
equal
sign
symbol
times
factor
product
horizontal
vertical
parentheses
share
fair
quotient

Materials

counters
overhead counters
number cube

Transparencies:

Equal Groups
Hershey Kisses
Dividing Groups

Transparencies and Wall Posters:

All About Multiplication
All About Division
Operation Relationships

Student Copies:

Working with Equal Groups
More Work With Groups
Multiplication Practice
Practice With Division
Fact Families With Multiplication and Division

Language Foundation

1. Put the prefix **multi-** on the blackboard. Tell students that when this prefix comes at the beginning of a word, it means "many or more than one". Give some examples, such as multicolored, multimedia, multivitamin, multinational, multilingual, and multipurpose. Ask students if they can think of other words like these. (multilink cubes)

Now, put the adjective **multiple** on the board. Explain that it means many. It is used to describe things. Give examples like multiple parts, multiple chairs, etc. Ask students for other examples. Remind students that on tests, they may have multiple-choice questions, meaning that they have many choices for the answer.

Tell students that the verb to **multiply** means to make many or to increase in number. In this lesson, they will put groups of things together to make many things. **Multiplication** is the process, or what we do when we multiply numbers.

2. **Divide** means to separate into parts. Instead of putting groups of things together, they will be taking groups apart and making equal groups in this lesson.

Explain that **share** is to give or receive a part of something. Draw a loaf of bread on the board, but don't divide it into slices. Ask students how you could share this loaf with the students in the class. Now, draw slices in the loaf and explain that you had to divide it into parts so that each student would get a part.

3. As you explain the **symbols** and **signs** for multiplication and division, remind students of the ones used in addition and subtraction. These are short ways of expressing the operation without using the words.

Mathematics Component

Note: Children begin to encounter informal multiplication and division situations in the early primary grades. For example, a story may introduce a group of three children who after baking cookies with their mother eat two cookies each. The question, "How many total cookies did the three children eat?" would come naturally from this story. These types of early counting experiences are the beginning steps which lead students to a sound understanding of multiplication and division. Concept development using **concrete** materials, **strategies** which promote memorization of basic facts, and finally work with **formal algorithms** must all be included if students are to attain mastery of these concepts.

Multiplication as Repeated Addition

1. To develop an understanding of what multiplication means, use the Equal Groups transparency and talk with students about these situations which show groups of the same size.
 - Point to the bicycles and say, "How many bicycles are in the picture?" (3) "how many wheels does each bicycle have?" (2)
 - Write "3 groups of 2" on the line below the bicycles.
 - Have one student come up to the overhead and count the total number of wheels as they point to each one. (6)
 - Ask students if there is another way to find the total number of wheels on the bicycles. Lead the class to understand that we can also add $2 + 2 + 2$. Discuss why this addition sentence represents the total number of wheels. (There are 2 wheels on the first bicycle, 2 on the second, and 2 on the third.)
 - Write " $2 + 2 + 2 = 6$ " on the addition sentence line. Point out to students that there are three bicycles and each has the same number of wheels (2)
 - Tell the class that another way to represent the total number of wheels is with multiplication. Explain that since there are "3 groups of 2" or " $2 + 2 + 2$ " we can write a multiplication sentence to represent the amount.
 - Record " $3 \times 2 = 6$ " on the multiplication line below the picture. Point to "x" as you say, "This is a multiplication sign and we say times. Three times two equals six."
 - Have students think of "times" as "groups of." Point to 3×2 as you say, "Three times two or three groups of two."
 - Repeat the same procedure for each of the other three pictures, filling in the following information for each one.

<u>Hands</u>	<u>Clover</u>	<u>Legs (Horses)</u>
2 groups of 5	4 groups of 3	2 groups of 4
$5 + 5 = 10$	$3 + 3 + 3 + 3 = 12$	$4 + 4 = 8$
$2 \times 5 = 10$	$4 \times 3 = 12$	$2 \times 4 = 8$

- Have students complete the activity sheets Working With Equal Groups and More Work With Groups.

2. Students work with a partner to complete this activity. Distribute a small bag of counters (approximately 25-30 to each pair of students. Each individual student will need a piece of paper and a pencil to record their work. You will also need overhead counters and an overhead or regular number cube.

- Tell students that they will use counters to represent multiplication problems.
- Toss a number cube and model making a group of that many counters. Have each pair of students use counters to do the same. (For example, tossing a 2 would require that 2 counters be placed on the overhead.)



- Toss the number cube a second time and model making that total number of groups, each one equal in size to the first group. Have students do the same. (For example, tossing a 3 would require making 3 groups of 2 counters - two in addition to the the first group!)



- Ask students to find the total number of counters. (6)
- Have students suggest a multiplication sentence that would represent these groups of counters. ($3 \times 2 = 6$) Write the sentence on the board and ask students to do the same on their paper.

$$3 \times 2 = 6$$

- Tell students that the two numbers being multiplied together are each called a factor and the answer to the multiplication sentence is called the product. Label each of these on the board and have students do the same.

$$\begin{array}{ccc} \text{factor} & \text{factor} & \text{product} \\ \swarrow & \searrow & \swarrow \\ & 3 \times 2 = 6 & \end{array}$$

- Review that the sentence they have written is in a horizontal position. Say, "Multiplication sentences may also be written in a vertical position." Write the vertical form on the board as shown and have students do the same.

$$3 \times 2 = 6$$

$$\begin{array}{r} 3 \\ \times 2 \\ \hline 6 \end{array}$$

- Ask a student to name the factors in the vertical problem. If he or she seems unsure, remind them that factors are the numbers being multiplied together. (3 and 2) Have another student name the product in the vertical problem. (6) Remind the class that the product is the answer to a multiplication sentence.
- Write a times sign (x) on the board and ask students what they think of when they see this sign. (They should think "groups of." We read the sign as "times.")
- Tell students that there is another way to write a multiplication sentence. Record this third method beside the first two on the board.

$$3 \times 2 = 6$$

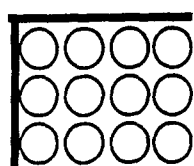
$$\begin{array}{r} 3 \\ \times 2 \\ \hline 6 \end{array}$$

parentheses

$$3 (2) = 6$$

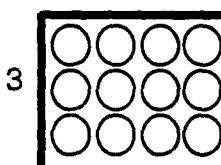
- Say, "This means the same as the other two and is read the same way - Three times two equals six."
 - Be sure that students can identify the factors (2 and 3) and the product (6) in each of the three problems.
 - Repeat the same procedure with students several more times by:
 - 1) tossing a number cube and placing that number of counters in a group,
 - 2) tossing the number cube again and representing that total number of groups,
 - 3) recording the multiplication sentence in three different ways (vertically, horizontally, and using parentheses.
 - 4) identifying the factors and the product.
 - When students are comfortable with the multiplication concept, have them complete the activity sheets Multiplication Practice.
 - A transparency master/wall poster, All About Multiplication, is provided for review and/or reinforcement of this concept.
3. The meaning of division may also be modeled for students using counters.
- Place the Hershey Kisses transparency on the overhead.
 - Read the problem aloud.
 - Underline the word share on the transparency. Discuss the concept of sharing. (Sharing means each person gets some.)
 - Choose a few Hershey Kisses from a bag and say, "Two students will share these chocolates." Give a student in the class one Hershey Kiss and give a second student three Kisses.
 - Ask the student with one Hershey Kiss if this was a good way to share the chocolates. Why? (The student hopefully will say no because the other student got more.)
 - Discuss the word "fair" and allow students to suggest a fair way to share the four Hershey Kisses. (Two chocolates each would be a "fair" way to share them.)

- Tell students that if each person gets the same amount, we are sharing **equally**.
- Read the problem a second time and help students focus on the question. (How many Kisses did Mrs. Delgado put in each bag?)
- Underline “the same number” and explain that Mrs. Delgado made sure that the children were **sharing equally**.
- Pick up a small number of transparent counters and say, “How many Hershey Kisses did Mrs. Delgado have for her children to share equally?” (12) Count and place 12 counters on the overhead.
- Ask students how we can use the counters to find the number of Kisses Mrs. Delgado put in each bag. Lead students to understand that we can **divide** the counters into 3 equal groups, one group for each child.
- Model dividing the counters into three equal groups as shown below.



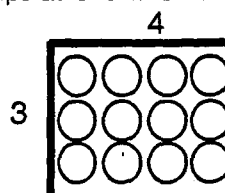
12

Dividing things into equal groups is called division. We use a special symbol for division.



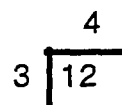
12

We divided the 12 counters into 3 **equal** groups.



12

There are 4 in each equal group. Mrs. Delgado put 4 Kisses in each bag.



We write twelve divided by three equals four.

- Tell students that there are **two** ways to record division. Write each of the two ways side-by-side as you say, “Twelve divided by three equals four.”

$$\begin{array}{r} 4 \\ 3 \overline{)12} \end{array}$$

or $12 \div 3 = 4$

- Point to the number 4 in each of the problems and explain that the answer to a division problem is called the **quotient**. Say, “Four is the quotient for this problem.”
- Once students are comfortable with the concept of division, work together as a class on the Dividing Groups transparency. (Follow pattern shown above.)
- Additional reinforcement is provided on the activity sheets Practice With Division.
- A transparency master/wall poster, All About Division, is included for review and/or reinforcement of this concept.

4. The ability to mentally reverse operations enables students to use their knowledge of one operation to develop an understanding of another.

- Review the inverse relationship between addition and subtraction (developed in the first objective of the operation unit) by writing the following problem on the board:

$$5 + 3 = 8$$

- Have students identify the “whole” in this number sentence. (8)
- Briefly review the relationship between addition and subtraction by reminding students that the whole (8) can be put together or taken apart. Lead students to understand that the number sentence above is putting the whole (8) together by adding $5 + 3$.
- Write the following number sentence on the board and ask a student to come up and fill in the blank.

$$8 - 3 = \underline{\hspace{2cm}} \text{ (5)}$$

- Remind students that this is one way to take the whole (8) apart.
- Tell students that equal groups of things can also be put together or taken apart. Explain that multiplication is a way to put together equal groups. For example, you can put together “3 groups of 2.” Write $3 \times 2 = 6$ on the board.
- Place 6 counters on the overhead and say, “I will divide the 6 counters (take them apart) into 2 equal groups.”



- Ask, “How many are in each group?” (3)
- Push the counters back together and say, “I will divide the counters into 3 equal groups.”



- Ask, “How many are in each group?” (2)
- Explain that multiplication and division are related, just like addition and subtraction. Use the Operation Relationships transparency to illustrate these relationships. This may also be enlarged and posted for student reference/review.
- Have students complete the Fact Families With Multiplication and Division activity sheet.

Equal Groups

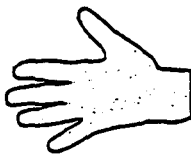
Bicycle Wheels



— equal groups of —

Addition sentence —

Multiplication Sentence —



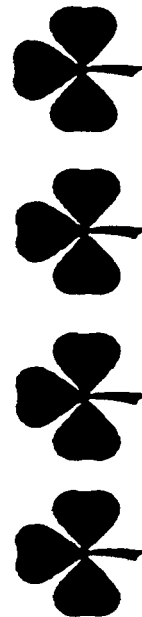
Hands

— equal groups of —

Addition sentence —

Multiplication Sentence —

Clovers



— equal groups of —

Addition sentence —

Multiplication Sentence —

Horses



— equal groups of —

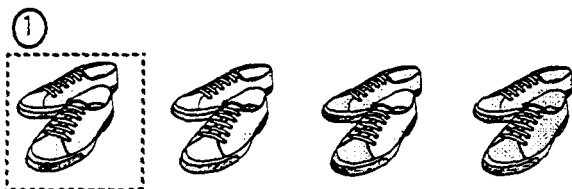
Addition sentence —

Multiplication Sentence —

Name: _____

Working With Equal Groups

For each picture: 1) tell how many groups 2) write a sentence for addition and multiplication



_____ groups of _____

Addition sentence _____

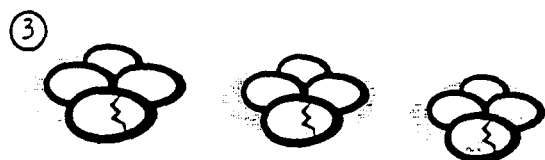
Multiplication sentence _____



_____ groups of _____

Addition sentence _____

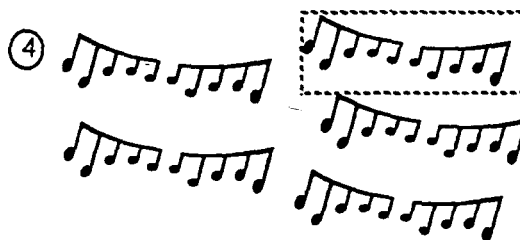
Multiplication sentence _____



_____ groups of _____

Addition sentence _____

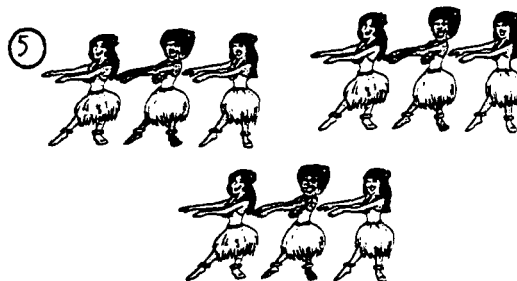
Multiplication sentence _____



_____ groups of _____

Addition sentence _____

Multiplication sentence _____



_____ groups of _____

Addition sentence _____

Multiplication sentence _____

⑥ DRAW YOUR OWN GROUPS

_____ groups of _____


Addition sentence _____

Multiplication sentence _____

Name: _____

More Work With Groups

Draw diamonds  to show the groups. Write the addition and multiplication sentences.

Example:		$4 + 4 = 8$	$2 \times 4 = 8$
	2 groups of 4		

- | | | |
|----|--|--|
| 1) | <div style="border: 1px solid black; width: 300px; height: 30px; margin: 5px 0;"></div> <div style="text-align: center;">5 groups of 2</div> | Addition _____
Multiplication _____ |
| 2) | <div style="border: 1px solid black; width: 300px; height: 30px; margin: 5px 0;"></div> <div style="text-align: center;">3 groups of 6</div> | Addition _____
Multiplication _____ |
| 3) | <div style="border: 1px solid black; width: 300px; height: 30px; margin: 5px 0;"></div> <div style="text-align: center;">2 groups of 7</div> | Addition _____
Multiplication _____ |

Matching - Draw a line to the correct addition or multiplication sentence. Find the answer.

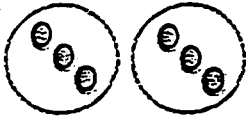
- | | |
|-------------------|--|
| 1) 5 groups of 3 | $9 \times 3 = \underline{\hspace{2cm}}$ |
| 2) 2 groups of 7 | $2+2+2+2 = \underline{\hspace{2cm}}$ |
| 3) 3 groups of 6 | $5 \times 3 = \underline{\hspace{2cm}}$ |
| 4) 4 groups of 2 | $8 + 8 = \underline{\hspace{2cm}}$ |
| 5) 8 groups of 5 | $3 \times 6 = \underline{\hspace{2cm}}$ |
| 6) 5 groups of 10 | $5 \times 10 = \underline{\hspace{2cm}}$ |
| 7) 9 groups of 3 | $5+5+5+5+5+5+5+5+5 = \underline{\hspace{2cm}}$ |
| 8) 2 groups of 8 | $2 \times 7 = \underline{\hspace{2cm}}$ |

Name: _____


Multiplication Practice

Write a multiplication sentence for each picture.
Write the number of groups as the first factor.

Example:



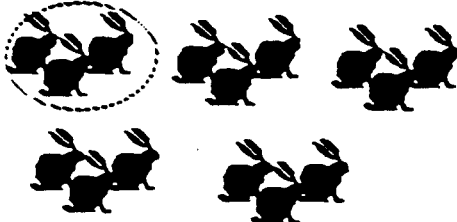
2 groups of 3
 $2 \times 3 = 6$

1) 

_____ groups of _____
_____ x _____ = _____

2) 

_____ groups of _____
_____ x _____ = _____

3) 

_____ groups of _____
_____ x _____ = _____

Solve each problem using a multiplication sentence.

1) How many legs are there?

_____ x _____ = _____



2) How many tails are there?

_____ x _____ = _____



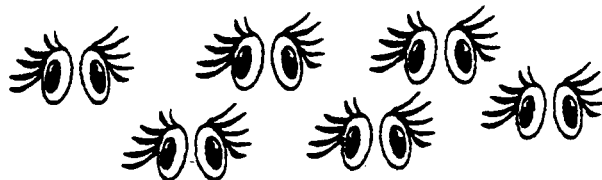
3) How many **wheels** are there?

_____ x _____ = _____



4) How many **eyes** are there?

_____ x _____ = _____



Find the product. Use cubes to model equal groups if you need help.

Think:
2 groups of 4

2
x 4
—
8

1)
$$\begin{array}{r} 3 \\ \times 4 \\ \hline \end{array}$$

2)
$$\begin{array}{r} 2 \\ \times 5 \\ \hline \end{array}$$

3)
$$\begin{array}{r} 1 \\ \times 4 \\ \hline \end{array}$$

4)
$$\begin{array}{r} 7 \\ \times 2 \\ \hline \end{array}$$

5)
$$\begin{array}{r} 5 \\ \times 1 \\ \hline \end{array}$$

6)
$$\begin{array}{r} 6 \\ \times 2 \\ \hline \end{array}$$

7)
$$\begin{array}{r} 3 \\ \times 5 \\ \hline \end{array}$$

8) $4 \times 4 = \underline{\hspace{2cm}}$

9) $8 \times 3 = \underline{\hspace{2cm}}$

10) $9 \times 2 = \underline{\hspace{2cm}}$

Use cubes to model and solve these problems.

Can you write a multiplication sentence for each problem? Check Yes or No?

$6 + 6 + 6$

Equal groups ☐ yes ☐ no

Multiplication sentence? ☐ yes ☐ no

_____ x _____ = _____

$2 + 4 + 9$

Equal groups ☐ yes ☐ no

Multiplication sentence? ☐ yes ☐ no

_____ x _____ = _____

$1 + 7 + 5$

Equal groups ☐ yes ☐ no

Multiplication sentence? ☐ yes ☐ no

_____ x _____ = _____

$2 + 2 + 2$

Equal groups ☐ yes ☐ no

Multiplication sentence? ☐ yes ☐ no

_____ x _____ = _____

All About Multiplication

$$\begin{array}{ccccc} 5 & \times & 3 & = & 15 \\ \uparrow & & \uparrow & & \uparrow \\ \text{Factor} & & \text{Factor} & & \text{Product} \end{array}$$

Say: Five times three equals fifteen.

Ways to Write Multiplication:

①

$$\begin{array}{c} 5 \times 3 = 15 \\ \uparrow \\ \text{times sign} \end{array}$$

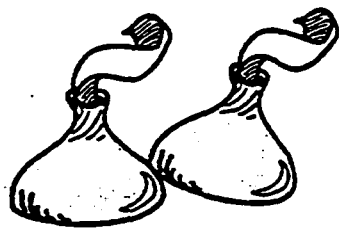
②

$$\begin{array}{r} 5 \\ \times 3 \\ \hline 15 \end{array}$$

③

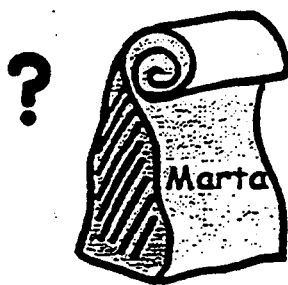
$$\begin{array}{c} 5(3) = 15 \\ \uparrow \quad \uparrow \\ \text{parentheses} \end{array}$$



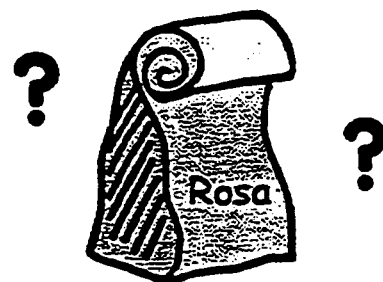


Hershey Kisses

Mrs. Delgado was making lunches for her 3 children to take to school. She had 12 Hershey Kisses for the children to share in their lunches. She put the same number of Kisses in each bag. How many candies did Mrs. Delgado put into each bag?





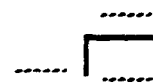
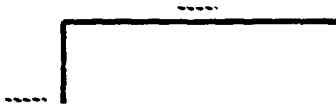


Dividing Groups

Mrs. Wong has ten gold stars. She wants to divide the stars equally among five students. How many stars will each student get?



Divide the 10 stars into 5 equal groups.



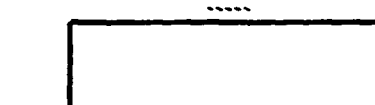
There are 2 stars in each group

Write 10 divided by 5 equals 2.

Mohamed had nine dog bones to give to his three dogs. How many bones did each dog get?



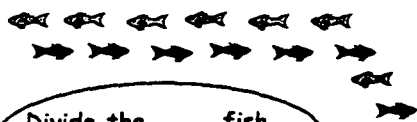
Divide the ____ bones into ____ equal groups.



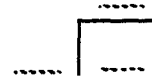
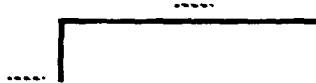
There are ____ bones in each group

Record the division problem

Nina has fourteen fish. She needs to put them in two tanks. How many fish will be in each tank?



Divide the ____ fish into ____ equal groups.



There are ____ fish in each group.

Record the division problem

Name: _____

Practice With Division



Remember - Use division when you want to separate things into groups of equal size!

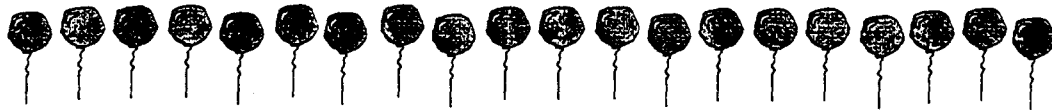
How many groups are there? Circle the groups. Write the number.



1) How many groups of 3 are there in 15? _____ groups



2) How many groups of 2 are there in 6? _____ groups



3) How many groups of 5 are there in 20? _____ groups

How many in each group? Draw cubes ■ to show groups. Write the number.

■ ■	■ ■	■ ■	6 Total 3 Groups	<u>2</u> cubes in each group
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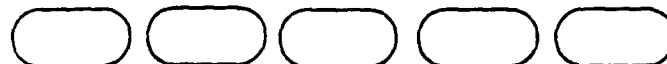
8 Total 2 Groups

_____ cubes in each group



24 Total 4 Groups

_____ cubes in each group

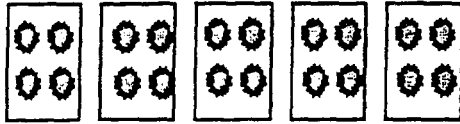


15 Total 5 Groups

_____ cubes in each group

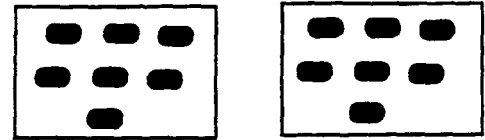
Solve. Use the pictures to help you.

1)



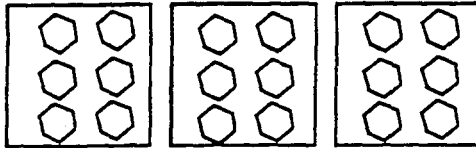
How many in all? _____
 How many groups? _____
 How many in each group? _____
 $20 \div 5 =$ _____

3)



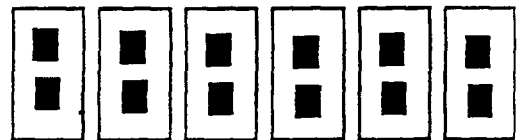
How many in all? _____
 How many groups? _____
 How many in each group? _____
 $14 \div 2 =$ _____

2)



How many in all? _____
 How many groups? _____
 How many in each group? _____
 $18 \div 3 =$ _____

4)



How many in all? _____
 How many groups? _____
 How many in each group? _____
 $12 \div 6 =$ _____

Find the quotient. Use models to help.

1) $4 \div 2 =$ _____

4) $16 \div 8 =$ _____

2) $7 \div 7 =$ _____

5) $9 \div 1 =$ _____

3) $30 \div 6 =$ _____

6) $12 \div 3 =$ _____

1) $10 \overline{) 20}$

4) $3 \overline{) 18}$

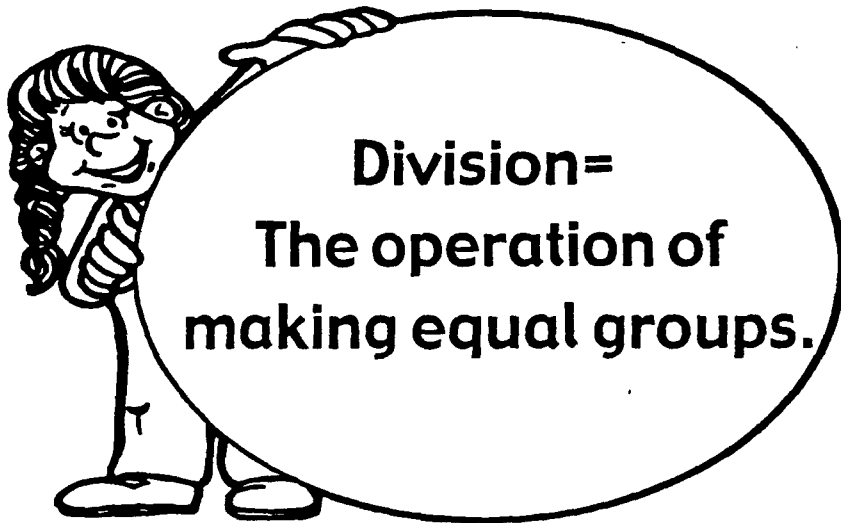
2) $2 \overline{) 6}$

5) $1 \overline{) 5}$

3) $4 \overline{) 16}$

6) $5 \overline{) 25}$

÷ All About Division }



15	÷	3	=	5
↑	↑	↑	↑	↑
dividend	divided by	divisor	equals	quotient

	5	← quotient
divisor →	3	
) 15	← dividend

Operation Relationships

Addition

Subtraction

$$4 + 3 = 7$$

$$7 - 3 = 4$$

$$3 + 4 = 7$$

$$7 - 4 = 3$$

Multiplication

Division

$$4 \times 5 = 20$$

$$20 \div 4 = 5$$

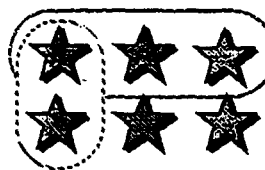
$$5 \times 4 = 20$$

$$20 \div 5 = 4$$

Name: _____

Fact Families With Multiplication and Division

Use multiplication to put groups together.
Use division to divide into equal size groups.




$$3 \times 2 = 6$$

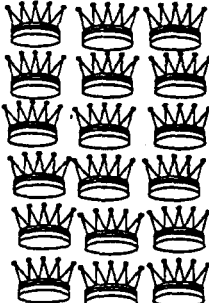
$$2 \times 3 = 6$$

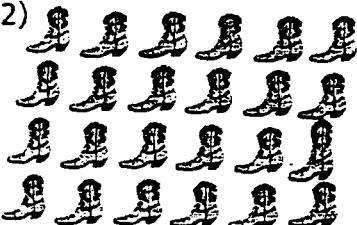
$$6 \div 3 = 2$$

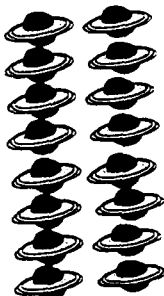
$$6 \div 2 = 3$$

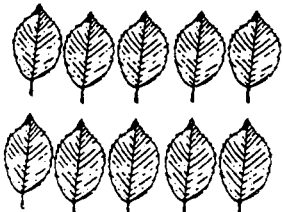
Complete the fact families.

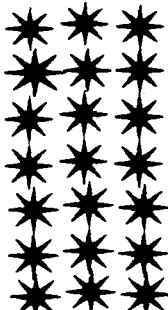
1)  $4 \times 3 = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}}$

4)  $3 \times 6 = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}}$

2)  $6 \times 4 = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}}$

5)  $2 \times 8 = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}}$

3)  $5 \times 2 = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}}$

6)  $3 \times 7 = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}}$

Objective 2: Use manipulatives to explore multiplication using arrays. Recognize prime, composite, and square numbers.

Vocabulary

array
row
column
by
times
square
prime numbers
composite numbers
square numbers
factor
product

Materials

counters (20/pair of students)
egg carton
inch tiles (bagged)
transparent inch tiles
centimeter graph paper
transparent graph paper
crayons or markers

Transparencies:

What is an array?
Building Arrays
Building Arrays Answer Key

Student Copies:

Working With Arrays
Building Arrays
Prime, Composite, and Square Numbers
Using Arrays with Multiplication and Division
More Fun with Arrays!

Language Foundation

1. Explain to students that the preposition **by** is used in multiplication to mean **times**, and it is represented by the same symbol for times.

Multiply 18 **by** 6.

Multiply 18 **times** 6.

Multiply 18 **x** 6.

These multiplication sentences all mean the same.

2. It should be pointed out when appropriate that **by** takes on a different meaning in division. Divide 18 by 6. This means that we should separate 18 into equal groups of 6.

Divide 18 by 6.

18 (is) divided by 6.

Divide 6 into 18.

$18 \div 6$

All of these division sentences translate to the same algorithm. The sentences cannot be translated literally.

Transferring the word sentence into the algorithm can be difficult for some students, especially if students are used to a different algorithmic construction.

$$\begin{array}{r} 18 \overline{) 6} \end{array}$$

Vietnam,
Cambodia, Laos
Pakistan, Iran
Armenia, Russia

$$\begin{array}{r} 18 \overline{) 6} \end{array}$$

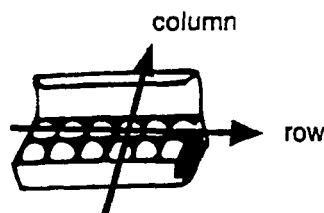
El Salvador

Notice that these constructions lend themselves to a literal translation of our division sentences.

3. Set the stage for prime and composite numbers. Relate the word **prime** to other cognates like primary. Lead students to think of one. Relate **composite** to composition, which is made up of more than one paragraph or part.

Mathematics Component

Note: The repeated addition method of multiplication taught in objective 1 provides an easy introduction to multiplication for most children, since it builds on the students' understanding of addition. The array method introduced in this objective will further strengthen the students' understanding of multiplication and lay a foundation for multiplying larger values. Instead of placing a group of objects into equal groups, students will place them into an **array**. An array is a systematic arrangement of items with equal rows and equal columns. An egg carton with two equal rows and six equal columns is an example of an array.



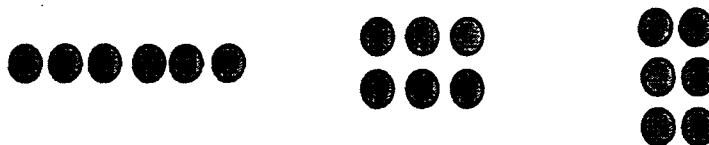
Multiplication Using Arrays

1. Place the transparency What is an array? on the overhead. Distribute 20 counters to each pair of students.
 - Point and tell students that each of the groups in box 1 forms an **array**. Each of the groups in box 2 do not form an array.
 - Point to box 3 and ask students which groups they think form an array. (a and d)
 - Lead students to understand that an array is a pattern where each **row** has the same number of items and each **column** has the same number of items. Point out that rows go across (horizontally) and columns go up and down (vertically).
 - Have students think of things in the world around us that come in even rows and columns. (Boxes of crayons, packages of pencils, watercolors, eggs in a carton, buttons on a telephone, muffin pans, tomatoes in a package, etc.)
 - Ask students to explain what an array is as you fill in the definition at the bottom of the transparency page. (An array has the same number in each row and the same number in each column.)
 - Make two different arrays on the overhead - one with 7 counters and the other with 11. (Note: These arrays can only be made with one row because two rows would end up with different numbers in each row.)

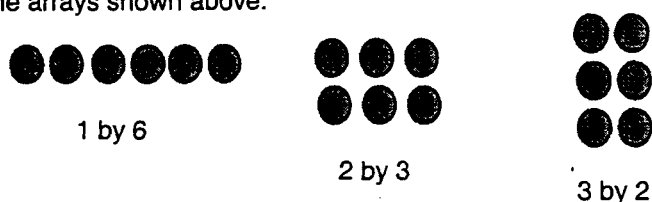


- Ask students how the two arrays are alike and how they are different. (They are alike because they each have only one **row**. They are different because they each have a different number in their row.)

- Make the following arrays on the overhead.



- Ask students how these arrays are alike and how they are different. (They are alike because they all have a total of 6 counters. They are different because they each have a different number of **rows and columns.**)
- Tell students that we name arrays by looking at the number of rows and columns. Explain that we count the number of rows first and then the number of columns. Discuss the correct name for each of the arrays shown above.



- Point to the first array and say, "This is a 1 by 6. We can also name the array using a number sentence. The row is repeated one time and there are six in the row. We can say one times six." Write 1×6 below the first array.
- Write number sentences for the other two arrays, reminding students that we name the number of rows first! In the middle array the row repeats two times with three in each row so we write 2×3 . In the last array the row repeats three times and there are two in each row so we write 3×2 .
- Have students build and name several more arrays until they are comfortable with this concept.
- Have students complete the activity sheet Working With Arrays.

For this activity, students will work with a partner. Distribute a small bag with 30 inch tiles and the activity sheet Building Arrays to each pair of students. Use a transparency copy to model and share results with the class.

- Place the Building Arrays transparency on the overhead. Put 2 separate transparent tiles in the workspace at the bottom and ask students if we can use these to make an array. (Yes, we can make a 1×2 array.)
- Tell students that for this activity you will push the tiles together to form an array.
- Model creating a 1×2 array on the overhead and point out the number of rows and columns as shown below.



- On the chart point to the first column labeled Number of Tiles.

- On the chart, point to the first column labeled Number of Tiles.
(Note: Skip 1 tile for right now. We will come back to that later.)
- Fill in the name of this array beside the number 2 as shown below. Say, "One times two equals 2."

Number of Tiles	1 by ?	2 by ?	3 by ?	4 by ?	5 by ?
1					
2	1 x 2				
3					

- Ask students if there is another way to build an array using 2 tiles. (Yes) Model and name the second array as shown below.



1 x 2

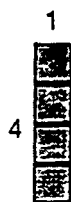
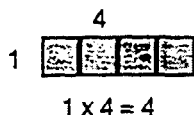


2 x 1

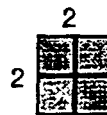
- Write 2 x 1 on the chart as you say, "Two times one equals 2."

Number of Tiles	1 by ?	2 by ?	3 by ?	4 by ?	5 by ?
1					
2	1 x 2	2 x 1			
3					

- Ask students if we can use 2 tiles to build a different array. (No)
- Remove the array from the workspace and replace it with 3 separate tiles. Repeat the same procedure asking students to build and name as many arrays as possible using the three tiles. (There is a 1 x 3 and a 3 x 1.)
- Fill in the name of these arrays in the appropriate place on the chart as you say, "One times three equals three. Three times one equals three."
- Repeat this same procedure with 4 tiles. (Students will be able to make a 1 x 4 array, a 4 x 1 array, and a 2 x 2.)



4 x 1 = 4



2 x 2 = 4

- Point to the third array and tell students that this is a special array called a **square**. A square has the same number of rows and columns.

- Fill in the chart in the appropriate columns as you say, "One times four equals four, four times one equals four, and two times two equals four."

Number of Tiles	1 by ?	2 by ?	3 by ?	4 by ?	5 by ?
1					
2	1 x 2	2 x 1			
3	1 x 3	3 x 1			
4	1 x 4	2 x 2		4 x 1	
5					

- Have students work with a partner to complete the chart. Tell them to work with their partner to find as many arrays as possible for each number of tiles through 20. (Note: Be sure to explain to the students that we are only recording arrays up to "10 by ?" even though there may be others. Other arrays may be discussed as you go over the answers.)
- When students have finished, go over the chart together, letting different students come up and fill in the array names on the transparency. If there is confusion over some arrays, build them on the overhead. (See the completed chart in the answer key to this objective.)
- Now that students are comfortable with the concept of rows and columns, go back to the beginning of the chart where students are asked to build an array using 1 tile. (1 by ?)
- Ask students to name the number of rows and columns using 1 tile. Fill in the name of the array in the appropriate place on the chart. (1 x 1)

Analysis of Array Chart (Prime, Composite and Square numbers)

- Have students look at the numbers on the chart which only have two arrays listed. Ask students what they notice about these numbers. (The only **factors** of these numbers are 1 and the number itself. For example, for the number 2 there are two arrays (1 x 2 and 2 x 1). The only factors of 2 are 1 and 2.)
 - Tell students that these numbers have a special name. Say, "They are called **prime numbers**. Prime numbers are numbers which only have two factors - the number itself and 1."
 - Make a list of all of the prime numbers on the chart. (2, 3, 5, 7, 11, 13, 17, 19)
 - Ask students what they think the next two prime numbers would be. (23, 29...)
 - Explain that all other numbers which have more than two arrays listed are called **composite numbers**. Make a list of all of the composite numbers which are on the chart. (4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20)
 - Ask students what they think the next two composite numbers would be. (21, 22...)
- (Note: The number 1 is neither prime nor composite.)

- Ask students to look at the completed chart and see if they can name any arrays that are squares. Remind students that squares have the same number of rows and columns. (2×2 , 3×3 , 4×4 ...)
- Lead students to understand that the numbers which can be represented with a square are called **square numbers**.

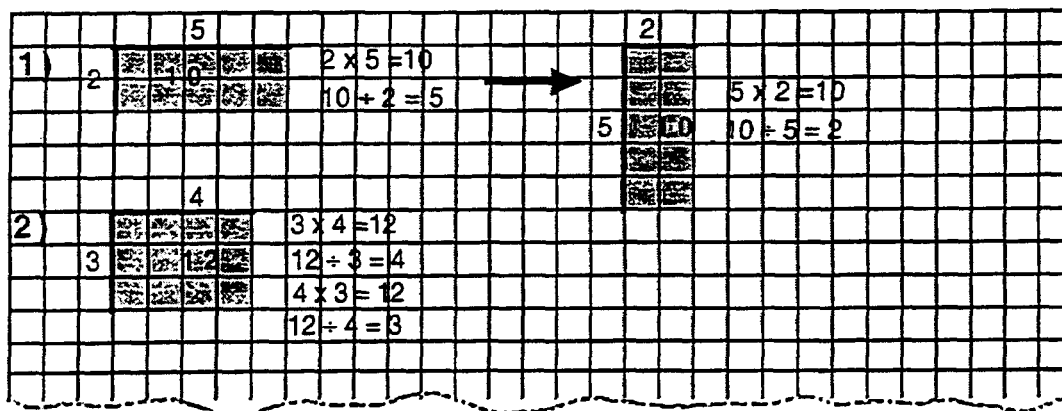
$$\begin{array}{l}
 1 \times 1 = 1 \\
 2 \times 2 = 4 \\
 3 \times 3 = 9 \\
 4 \times 4 = 16
 \end{array}
 \begin{array}{l}
 \diagup \\
 \diagdown
 \end{array}
 \begin{array}{l}
 1, 4, 9, \text{ and } 16 \text{ are square numbers}
 \end{array}$$

- Have students name the **factors** that produce square **products**. (For example, 2 and 2 are the factors of the product 4.)
- Ask students to talk with a partner and think about other square numbers. Have students suggest other squares and tell the factors for each one. (25, 36, 49...)
- Have students explain the pattern they notice in square numbers. (The two factors are the same.)
- Ask students how many square numbers could be created using this pattern. Lead them to understand this pattern could go on forever.
- Have students work on the activity sheet Prime, Composite, and Square Numbers.

Distribute a piece of centimeter graph paper to each student. Place a transparency copy of the graph paper on the overhead.

- Draw a 2 by 5 array in the top left-hand corner of the transparency. Have students do the same.
- Ask students to label the number of rows, columns, and total blocks as you model on the transparency. (See grid paper below.)
- Have students give a number sentence which names this array. Write the number sentence beside the array and have students do the same. ($2 \times 5 = 10$)
- Ask students to name the related division sentence which could be written if the whole is taken apart. Drawing a division symbol around the array may help students.
- Write the division sentence as shown below and have students do the same. ($10 \div 2 = 5$)
- Tell students that you will now turn the array around so that it is called a 5 by 2. Fill in a 5 by 2 area beside the first array and have students do the same.
- Have students suggest the two related number sentences which may be written for this array and write them on the transparency. Have students do the same. ($5 \times 2 = 10$ and $10 \div 5 = 2$)
- Point out that even though the array has been turned around, the factors and the product are the same. Lead students to understand that for each set of factors and its product, there are four related number sentences.
- Ask students to color in a 3 x 4 array as you model on the overhead. Label the number of rows, columns, and total blocks.

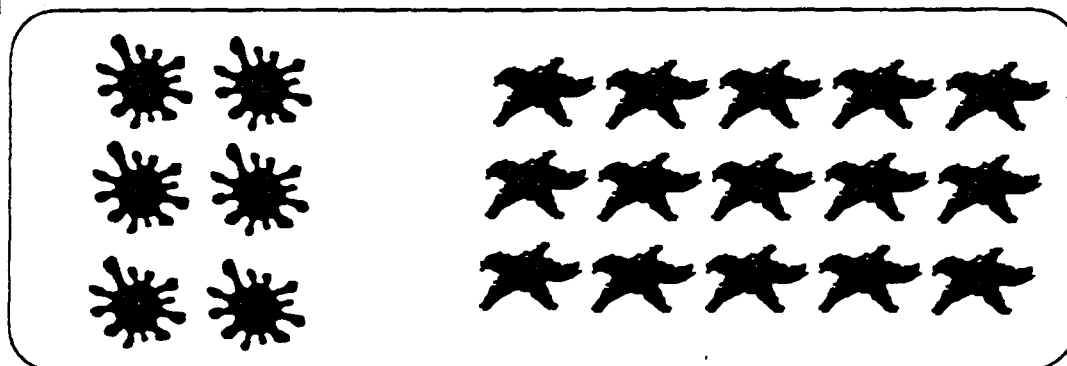
- Have students work with a partner to write the four related number sentences. Ask students to share responses as you record them on the transparency. (See chart that follows.)



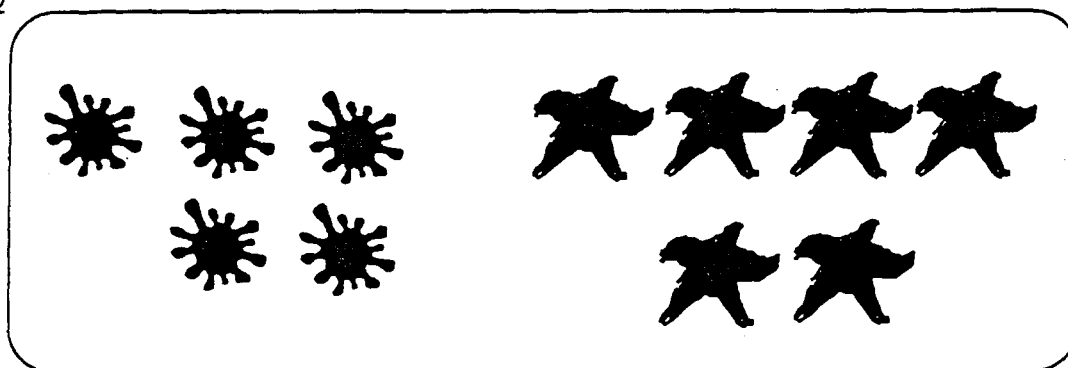
- Make a list of the following arrays on the board. Ask students to draw and label each array and then write four related number sentences for each one.
 - 3) 6 by 2
 - 4) 1 by 5
 - 5) 3 by 2
 - 6) 3 by 7
- When everyone is finished, check answers by letting students come up and fill in the information on the overhead.
- The activity sheets Using Arrays with Multiplication and Division and More Fun with Arrays! are provided for additional practice.

What is an array?

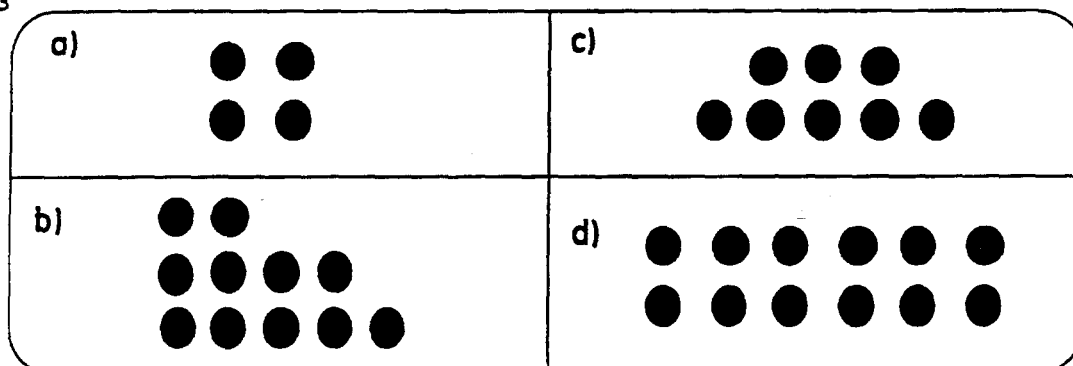
Box 1



Box 2



Box 3



Name: _____

Working with Arrays

Look at each array. Count how many rows and how many columns.

1)



_____ rows

_____ columns

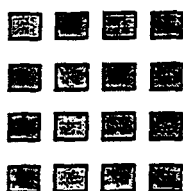
2)



_____ rows

_____ columns

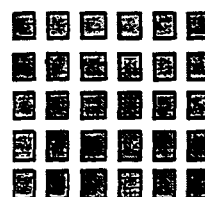
3)



_____ rows

_____ columns

4)



_____ rows

_____ columns

Draw an array and solve.

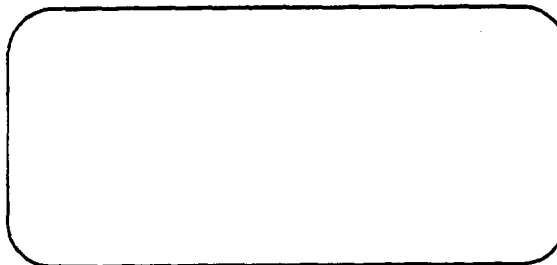
7 rows of cubes
2 columns of cubes
_____ cubes in all



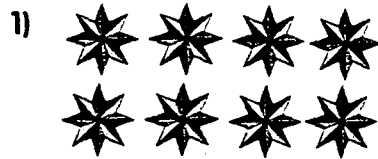
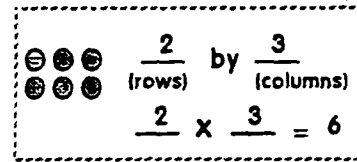
4 rows of circles
6 columns of circles
_____ circles in all



- Draw your own array.
- Write the number of rows and columns.
- Solve.



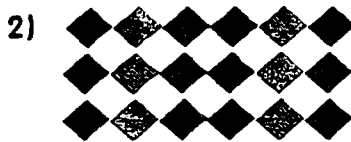
Name each array. Tell how many in all.



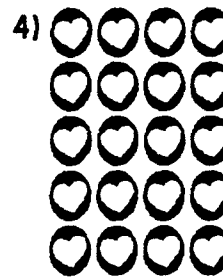
_____ by _____
_____ x _____ = _____



_____ by _____
_____ x _____ = _____

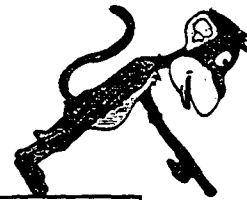


_____ by _____
_____ x _____ = _____



_____ by _____
_____ x _____ = _____

Use the ARRAY to fill in the answers on the chart.



How many rows?	How many columns?	How many in all?
A)		
R)		
R)		
A)		
Y)		

A A A A A A A
A A A A A A A
A A A A A A A

R R R R

R R R R R
R R R R R

A A A A A A A A
A A A A A A A A
A A A A A A A A

Y Y Y
Y Y Y
Y Y Y

Building Arrays

Number of Tiles	1 by ?	2 by ?	3 by ?	4 by ?	5 by ?	6 by ?	7 by ?	8 by ?	9 by ?	10 by ?
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										
11										
12										
13										
14										
15										
16										
17										
18										
19										
20										

Building Arrays Answer Key

Number of Tiles	1 by ?	2 by ?	3 by ?	4 by ?	5 by ?	6 by ?	7 by ?	8 by ?	9 by ?	10 by ?
1	1 x 1									
2	1 x 2	2 x 1								
3	1 x 3		3 x 1							
4	1 x 4	2 x 2		4 x 1						
5	1 x 5				5 x 1					
6	1 x 6	2 x 3	3 x 2			6 x 1				
7	1 x 7						7 x 1			
8	1 x 8	2 x 4		4 x 2				8 x 1		
9	1 x 9		3 x 3						9 x 1	
10	1 x 10	2 x 5			5 x 2					10 x 1
11	1 x 11									
12	1 x 12	2 x 6	3 x 4	4 x 3		6 x 2				
13	1 x 13									
14	1 x 14	2 x 7					7 x 2			
15	1 x 15		3 x 5		5 x 3					
16	1 x 16	2 x 8		4 x 4				8 x 2		
17	1 x 17									
18	1 x 18	2 x 9	3 x 6			6 x 3			9 x 2	
19	1 x 19									
20	1 x 20	2 x 10		4 x 5	5 x 4					10 x 2

Name _____

Prime, Composite and Square Numbers

Put the numbers in the box in the correct group: prime, composite or square.

(Remember: some composite numbers are also squares!)

5	4	36	22
		16	
7	30	11	
			18
10		19	25

Prime

Composite

Square

Pick one number from each group and draw an array.


Write the number you used under the box.


Explain prime, composite and square numbers.


Name: _____

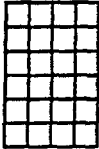
Using Arrays with Multiplication and Division

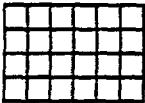
Draw and label each array. Write a multiplication and division sentence for each array.

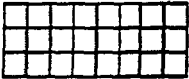
Example  $2 \times 2 = 4$
 $4 \div 2 = 2$

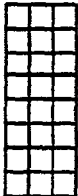
1)  _____


 _____


2)  _____

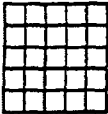
 _____

3)  _____

 _____

4)  _____

 _____

5)  _____

What will
happen
here? _____

Name: _____

More Fun With Arrays !

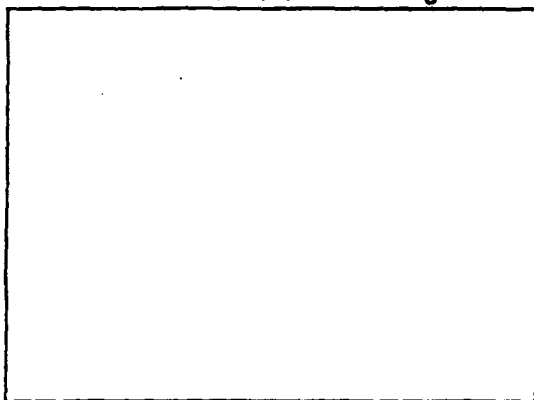
Draw the array in a different way.



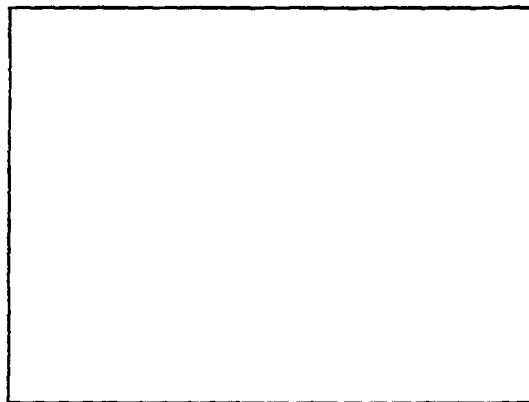
Can you make a greater number of arrays with 8 circles or 9 cubes ?

Draw arrays in the boxes to prove your answer.

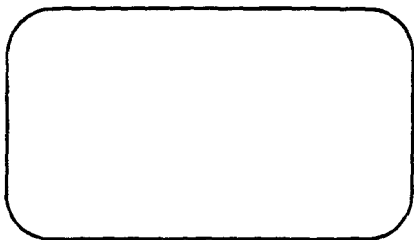
8 Circles



9 Cubes



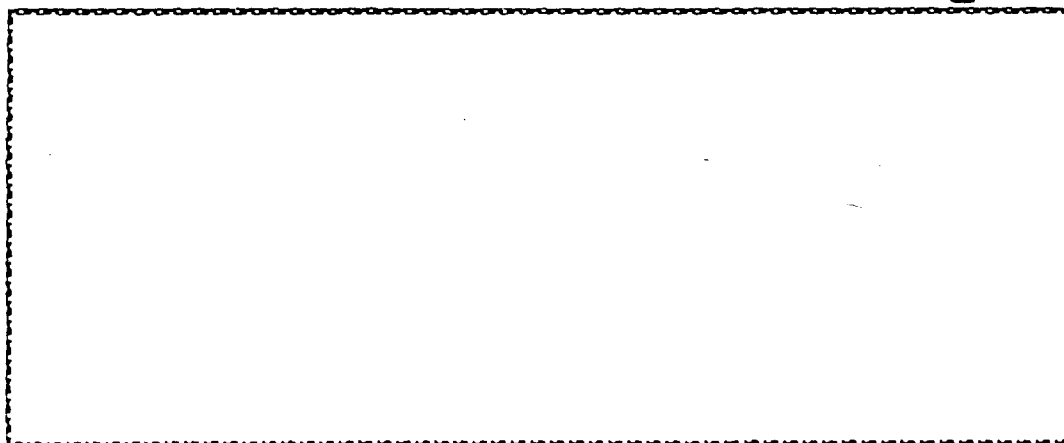
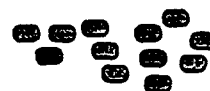
Draw an array with three rows. Name the array. How many objects are there in all?



_____ by _____

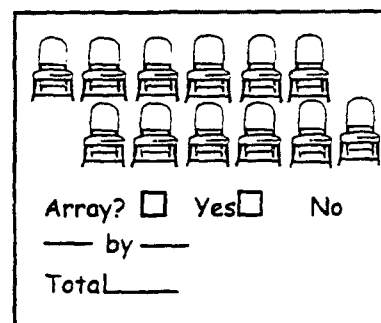
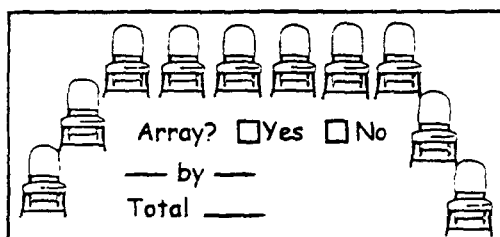
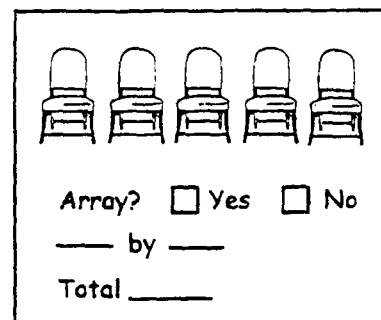
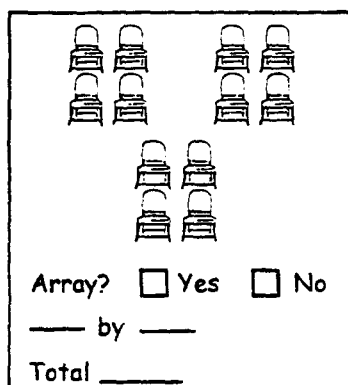
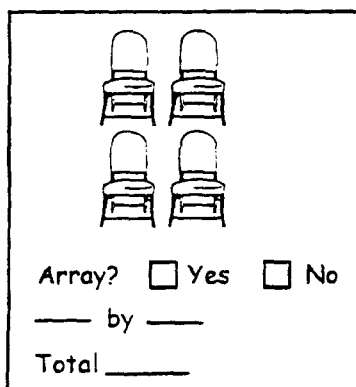
_____ = _____

Using 12 counters, how many different arrays can you make?
Draw them all. Label your arrays.



You are having a family dinner at your house on Saturday. You must have chairs for everyone to sit on. These pictures show different ways to arrange the chairs.

- 1) Which chairs are in an array?
- 2) Label each array.
- 3) Find the total for each group of chairs.



Objective 3: Mentally compute multiplication facts through nines. Recognize the use of properties and strategies:

- commutative
- zero
- identity
- nines
- doubles
- fives
- skip counting

Vocabulary

factor
product
commutative
identity

Materials

transparent nickels

Transparencies:

centimeter grid paper (TR)
Multiplication Properties (Obj. 4)
Doubles (Add/Sub Obj. 1)

Student Copies:

What's The Order?
Let's Change Places!
Multiplying With Zeros and One
More Practice With 0 and 1
Multiplication Doubles
Multiplication With Fives

Language Foundation

1. A review of the strategies introduced in addition and subtraction objective 2 would be appropriate before or during this lesson.
2. Remind students that **factors** are numbers multiplied to get a product. The answer to a multiplication problem is the **product**.

$$5 \times 2 = 10 \text{ (product)}$$



(factors)

3. Introduce the concept of **identity** in #3 of this lesson. Students have ID numbers while they are in school. These ID numbers identify the student. These numbers remain the same as long as the student remains in the school system, even though things such as age, weight, height, or hair styles may change as they progress through the school years. Their identity does not change.

Mathematics Component

Note: Once students are comfortable with the meaning of multiplication, both as repeated addition and as arrays, they are ready to begin mentally mastering multiplication facts through nines. As with addition, mastery of the facts is made easier and more meaningful if they are approached through strategies rather than isolated memorization. This objective will introduce strategies needed for mastering the basic multiplication facts. Opportunities for students to share and discuss strategies and additional practice throughout the year will ensure students' success in making responses quick and automatic.

1. Commutative Property of Multiplication

In this activity, you will review the concept of an array and then connect that knowledge to an understanding of the Commutative Property of Multiplication.

- Place a transparency copy of centimeter grid paper on the overhead.
- Tell students that you will use Xs to fill in a 2 by 3 array. Mark the grid paper as shown below.

1)		X	X	X					
		X	X	X					

- Remind students that to name an array, we count the rows first and then the columns. A 2 by 3 array has two rows and three columns.
- Ask students what multiplication sentence can be written for this array. ($2 \times 3 = 6$) Record the sentence below the array on the overhead.
- Have a student come up and fill in a 3 by 2 array beside the first one. Ask a second student to write the multiplication sentence represented by that array.

		X	X	X		X	X		
		X	X	X		X	X		
						X	X		
		$2 \times 3 = 6$							
						$3 \times 2 = 6$			

- Ask students to tell how these arrays are the same and how they are different. (Same - they both use six total squares. different - they both have a different number of rows and columns.)
- Cut a small piece of 2 x 3 grid transparency and color it yellow. Fit it over the first array and then rotate it vertically to fit over the second array. lead students to understand that a second array using the same total number of squares can be made by turning the first array in a different position.

- Repeat this same procedure with a 3 by 4 array. Remind students that a new array with the same total number of squares can be made by turning the first array in a different position. (See illustration showing $3 \times 4 = 12$ and $4 \times 3 = 12$.)

1)		X	X	X		X	X		
		X	X	X		X	X		
		$2 \times 3 = 6$				X	X		
						$3 \times 2 = 6$			
		X	X	X	X		X	X	X
		X	X	X	X		X	X	X
		X	X	X	X		X	X	X
		$3 \times 4 = 12$					X	X	X
						$4 \times 3 = 12$			

- Pass out student copies of centimeter grid paper. Have students copy the arrays shown above.
- Write the following on the board.
 - 3) 2 by 5
 - 4) 4 by 6
 - 5) 7 by 3
- For each of the above (3-5) ask students to repeat the same procedure by:
 - filling in an array,
 - turning the array in a different position to create a second array with the same total number of squares, and
 - writing a multiplication sentence for each array.
- When students have finished, have them come up to the overhead and share responses by filling in arrays and multiplication sentences on the transparency.
- Review the words **factor** and **product** by pointing these out in a few of the multiplication sentences.
- Ask students if they see any patterns. (When you turn the array around, the answer is the same, but the numbers are in a different order.)
- Say, "This is called the Commutative Property of Multiplication. The Commutative Property of Multiplication says that when you change the order of the factors, the product stays the same."

- To tie this property to the Commutative Property for Addition introduced in Addition/Subtraction Objective 2, write the following on the board:

Commutative Property of Multiplication

$6 \times 4 = 24$	$4 \times 6 =$
$2 \times 7 = 14$	$7 \times 2 =$
$5 \times 4 = 20$	$4 \times 5 =$
$3 \times 6 = 18$	$6 \times 3 =$

- Have different students come up and replace the question mark in each sentence with a number.
- Write the following on the board:


Commutative Property of Addition

$2 + 3 = 5$	$3 + 2 = ?$
$9 + 1 = 10$	$1 + 9 = ?$
$5 + 6 = 11$	$6 + 5 = ?$
$4 + 5 = 9$	$5 + 4 = ?$

- Have different students come up and replace the question mark in each sentence.
- Ask the students how the Commutative Property for Multiplication and the Commutative Property for addition are alike. (Both properties say that the order of the numbers does not change the answer.)
- Have students complete the activity sheets What's the Order? and Let's Change Places.
- A transparency master/wall poster Multiplication Properties is provided for review in Obj. 4.

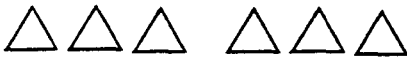
2. Zero Property

- Draw two groups of three triangles on the board as shown below. Ask students what multiplication sentence is represented by this picture. Fill in $2 \times 3 = 6$ (two groups of 3 equals six) under the picture.



 $2 \times 3 = 6$

- Now draw a vertical line and write 0×4 on the board as shown below. Ask if anyone can draw a picture to represent this multiplication sentence.



 $2 \times 3 = 6$

?

 $0 \times 4 =$

- Lead students to understand that 0×4 means 0 (no) groups of 4. Therefore, nothing would be drawn in the space provided. Point to the blank line after 0×4 and ask a student what the product of 0 and 4 would be. (0) Have a student fill in 0 as the answer.
- Say, "If I know that $0 \times 4 = 0$, what else do I know? Why?" (I know that $4 \times 0 = 0$ because the Commutative Property of Multiplication tells us that if the order of the factors changes, the product stays the same. Since $0 \times 4 = 0$, then $4 \times 0 = 0$.)
- Write the following problems on the board and ask students what is the same about all of them. $5 \times 0 =$, $0 \times 6 =$, $2 \times 0 =$, $8 \times 0 =$, $0 \times 7 =$, $0 \times 0 =$. (The product of all the problems is 0.)
- Say, "This is called the Zero Property of Multiplication. The Zero Property of Multiplication tells us that multiplying any number by 0 gives 0 as an answer."

3. Identity Property

- Write the word **identity** on the board as you say it aloud.
- Tell students that people each have their own identity. Write your name on the board under the word identity. Explain that you have many special qualities that make you the person that you are. No other person is exactly like you.
- Say, "I have my own identity."
- Do something to alter your appearance. For example, put on a hat or coat, take off your tie or scarf, etc.
- Ask students if your identity has changed. Lead them to understand that even though your appearance has changed, you are still the same person. Your identity has not changed, you are still Ms./Mrs./Mr. _____.
- Write the following problems on the board: $1 \times 6 =$, $1 \times 4 =$, $1 \times 3 =$, $1 \times 8 =$, $1 \times 5 =$.
- Ask students what is the same about all of these problems. (They all have a 1 as one of the factors.)
- Work through each of the above problems allowing students to explain to the class how they got the answer. They may use the repeated addition concept or the array method to demonstrate their thinking. For example in the first problem they may show one equal group of 6 items or fill in a 1×6 array.
- Record correct responses beside each of the problems.
- Have students talk to a partner about any patterns they see in these problems. (Each correct answer is the factor (number) in the problem which is not 1.
- Say, "Each time we multiply by 1, the number with the 1 stays the same. The number does not change even though we have multiplied it by 1. What would be a good name for this special rule? Why? Allow students to suggest ideas.
- Say, "This is called the **Identity Property of Multiplication**." Lead students to understand that this is a good name because multiplying any number by 1 gives the same number. For example, $1 \times 2 = 2$.
- Go back to each of the multiplication sentences above. Read the problem and ask what else

students know and why. For example for the first sentence say, "If I know that $1 \times 6 = 6$, what else do I know? Why?" (I know that $6 \times 1 = 6$ because the Commutative Property of Multiplication tells us that if the order of the numbers change, the answer stays the same.)

- Record each of the new multiplication sentences beside its partner on the board.
- For additional practice with the Identity Property, have students complete the activity sheets Multiplying with Zeros and One and More Practice With 0 and 1.
- The transparency master/wall poster, Multiplication Properties (in Objective 4) may be used for review throughout the year.

Doubles

- Place the transparency, Doubles, from OPR Addition/Subtraction Objective 2 on the overhead.
- Remind students that these are examples of **doubles** which we learned about earlier. Ask students what they remember about doubles. (Doubles are addition facts that use the same number two times.)
- Go to each of the pictures and write the addition double sentence that is illustrated.



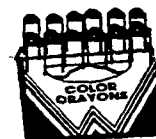
$$5 + 5 = 10$$



$$4 + 4 = 8$$



$$3 + 3 = 6$$



$$6 + 6 = 12$$

- Now go back to the picture of the hands and ask students if they can tell you a multiplication sentence for this picture. Allow time for students to discuss their thinking. Lead them to understand that since there are two equal groups of 5, we can write $2 \times 5 = 10$.
- Record $2 \times 5 = 10$ below the addition sentence $5 + 5 = 10$. Explain that there are two number sentences which represent the hands. Say, "Doubles can be written as an addition sentence or as a multiplication sentence."
- Repeat the same procedure for the remaining pictures, recording both the addition and multiplication sentence below each as you emphasize that there are two equal groups of each item.



$$5 + 5 = 10$$

$$2 \times 5 = 10$$



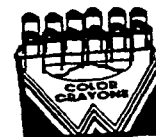
$$4 + 4 = 8$$

$$2 \times 4 = 8$$



$$3 + 3 = 6$$

$$2 \times 3 = 6$$



$$6 + 6 = 12$$

$$2 \times 6 = 12$$

- Say, "In the addition doubles, each number is written two times. What pattern do you see in the **multiplication doubles**?" (Multiplication doubles all use a 2 and one other number.)
- Say, "Multiplication doubles always have two equal groups."
- Draw the following on the board.



- Ask students, "Do these shapes represent doubles? Why?" (No, there are not two equal groups of squares.)
- Add one more square to the group which has 5 and repeat the same question. (Yes, there are two equal groups of squares.)



- Have students tell a partner a multiplication sentence which can be written to represent the two equal groups of squares. Write the sentence on the board. ($2 \times 6 = 12$)
- Write 7×2 on the board and ask students to tell a partner what they could draw to represent this double fact. Allow students to illustrate several ideas on the board.
- Further practice with the doubles strategy is provided on the activity sheets Multiplication Doubles.

5. Five Facts

- Place 9 transparent nickels on the overhead. Say, "How much money is this?" (45¢)
- Review with students and then model counting by 5s to get the sum.
- Write $5¢ + 5¢ + 5¢ + 5¢ + 5¢ + 5¢ + 5¢ + 5¢ + 5¢ = 45¢$. Say, "Could the sum be 47¢? Why?" (No, because the 5s facts end in only 0 or 5.)
- Tell students that you want to buy 7 pencils that each cost a nickel. Count out 7 nickels (1, 2, 3...) and remove the rest from the overhead.
- Say, "What multiplication sentence can I write to show how much 7 pencils cost?" Lead students to understand that adding 7 equal groups of 5¢ is the same as $7 \times 5¢ = 35¢$.
- Write 4×5 on the board. Ask students how you can think about nickels to find the answer to this multiplication sentence. (You can think of 4 nickels that are 5¢ each. Then you can count by 5s.)
- Model locating the amount which is equal to a nickel (5), underlining it in the problem, and then counting by 5s four times. (Note: Underlining the amount equal to a nickel helps some students clarify their thinking.)

$$\begin{array}{r} 4 \times 5 \\ \uparrow \\ \hline \end{array}$$

Count by 5s four times.

- Say, "What other ways could you find the answer for 4×5 ?" (You could draw an array with 4 rows and 5 columns, you could find the sum of $5 + 5 + 5 + 5$...)
- Write several other multiplication facts using 5s and allow students to practice reading the facts and then counting by 5s to find the product.
- Additional practice with the 5s facts is provided on the activity sheet Multiplication With Fives.

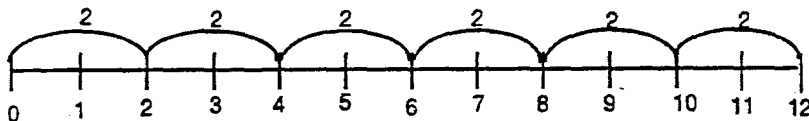
Additional Strategies

The strategies listed above are those which are easiest for all students to master. Allowing students to model and discuss other strategies promotes reflective thinking by all students. The following list provides additional strategies which may prove helpful to students in mastering basic multiplication facts.

Skip Counting

Students who are proficient at counting by 2s, 3s, 4s, or 5s may choose to skip count as a way to solve some multiplication problems. A number line is useful in modeling *skip counting*.

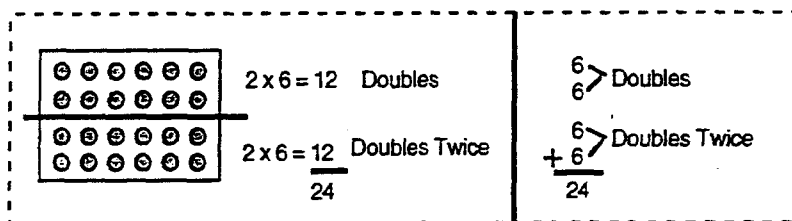
Example: $6 \times 2 = ?$ (Think: 2, 4, 6, 8, 10, 12)



Doubles Twice

This strategy is applicable to all multiplication facts with a 4 as one of the factors, either as the first or second factor.

Example: $4 \times 6 = ?$ or $6 \times 4 = ?$



Nines

The facts with 9 as a factor include the largest products, but are relatively easy to learn due to some interesting patterns. Two of these patterns combine to suggest one strategy which students may use:

- the tens digit of the product is always one less than the factor which is not 9
- the sum of the two digits in the product is always 9

Example: $9 \times 7 = ?$ \longrightarrow $9 \times 7 = \begin{array}{r} \text{one less} \\ \downarrow \\ 63 \\ \uparrow \uparrow \\ \text{add to 9} \end{array}$

A second pattern is evident when writing the 9s facts:

- the tens digits of the products go from 0 to 9 (top to bottom)
- the ones digits of the products go from 0 to 9 (bottom to top)

$\begin{array}{r} \text{0 to 9} \\ \downarrow \\ 9 \times 1 = 09 \\ 9 \times 2 = 18 \\ 9 \times 3 = 27 \\ 9 \times 4 = 36 \\ 9 \times 5 = 45 \\ 9 \times 6 = 54 \\ 9 \times 7 = 63 \\ 9 \times 8 = 72 \\ 9 \times 9 = 81 \\ 9 \times 10 = 90 \end{array}$ 0 to 9

Name: _____

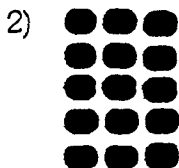
What's The Order?



This array shows 2 groups of 4.
The product of 2 and 4 is _____

Make an array to show 4 groups of 2.
The product of 4 and 2 is _____

What can you tell about 2×4 and 4×2 ? _____



This array shows _____ groups of _____
The product of _____ and _____ is _____

Turn the array in # 2. Draw it in the box.
This array shows _____ groups of _____
The product of _____ and _____ is _____

What can you tell about _____ \times _____ and _____ \times _____? _____

When we multiply, the Commutative Property of Multiplication tells us the order of the factors does not matter.



Find the product. Use the Commutative Property to help you.

1) $6 \times 2 =$ _____ so $2 \times 6 =$ _____

2) $3 \times 9 =$ _____ so $9 \times 3 =$ _____

3) $5 \times 1 =$ _____ so $1 \times 5 =$ _____

4) $7 \times 4 =$ _____ so $4 \times 7 =$ _____

1)
$$\begin{array}{r} 3 \\ \times 2 \\ \hline \end{array}$$

2)
$$\begin{array}{r} 5 \\ \times 6 \\ \hline \end{array}$$

3)
$$\begin{array}{r} 4 \\ \times 6 \\ \hline \end{array}$$

4)
$$\begin{array}{r} 2 \\ \times 3 \\ \hline \end{array}$$

5)
$$\begin{array}{r} 6 \\ \times 5 \\ \hline \end{array}$$

6)
$$\begin{array}{r} 6 \\ \times 4 \\ \hline \end{array}$$

7)
$$\begin{array}{r} 2 \\ \times 7 \\ \hline \end{array}$$

8)
$$\begin{array}{r} 9 \\ \times 3 \\ \hline \end{array}$$

9)
$$\begin{array}{r} 3 \\ \times 9 \\ \hline \end{array}$$

10)
$$\begin{array}{r} 7 \\ \times 2 \\ \hline \end{array}$$

Name _____

Let's Change Places !

Use the Commutative Property of Multiplication to find the products.

If you know

$5 \times 4 = 20$

$7 \times 5 = \underline{\hspace{2cm}}$

$2 \times 10 = \underline{\hspace{2cm}}$

$6 \times 3 = \underline{\hspace{2cm}}$

$1 \times 8 = \underline{\hspace{2cm}}$

Then you can do

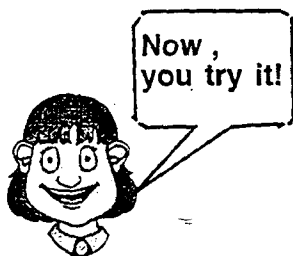
$\underline{4} \times \underline{5} = \underline{20}$

$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$



Using the Commutative Property, write 2 number sentences for the product.

The product is (6) so you can have

$$\begin{array}{l} 3 \times 2 = 6 \\ 2 \times 3 = 6 \end{array}$$

or

$$\begin{array}{l} 1 \times 6 = 6 \\ 6 \times 1 = 6 \end{array}$$

The PRODUCT is

18

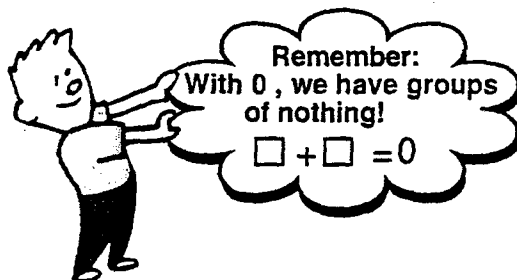
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24

Number Sentences

Name: _____

Multiplying With Zeros and One



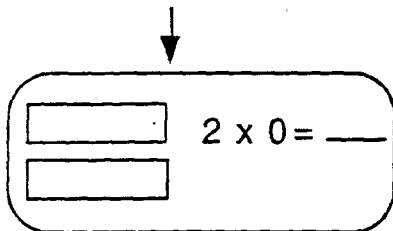
We have 4 groups of 0.
There is nothing in each group.
4 groups of nothing! $4 \times 0 = 0$
Zero Property of Multiplication

1) $0 + 0 + 0 + 0$
 $4 \times 0 = \underline{\quad}$

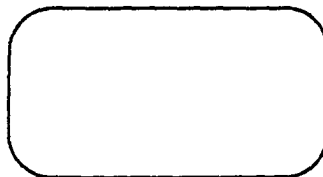
2) $0 + 0 + 0 + 0 + 0 + 0$
 $\underline{\quad} \times 0 = \underline{\quad}$

3) $0 + 0$
 $\underline{\quad} \times 0 = \underline{\quad}$

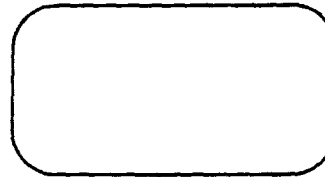
This picture shows two rows with 0 things in each row.
Write the multiplication sentence for each problem.



Show 3 rows with 0 things in each row.



Show 7 rows with 0 things in each row.

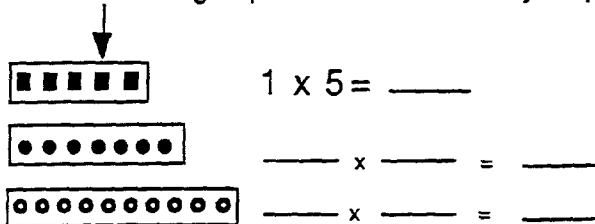


1) $1 + 1 + 1 + 1$
 $4 \times 1 = \underline{\quad}$

2) $1 + 1 + 1 + 1 + 1 + 1$
 $\underline{\quad} \times 1 = \underline{\quad}$

3) $1 + 1$
 $\underline{\quad} \times 1 = \underline{\quad}$

This picture shows one row with 5 things in the row.
We have **one** group of 5. Use the Identity Property.



$\begin{array}{r} 8 \\ \times 0 \\ \hline \end{array}$	$\begin{array}{r} 12 \\ \times 1 \\ \hline \end{array}$	$\begin{array}{r} 26 \\ \times 0 \\ \hline \end{array}$	$\begin{array}{r} 11 \\ \times 1 \\ \hline \end{array}$
$\begin{array}{r} 34 \\ \times 0 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 1 \\ \hline \end{array}$	$\begin{array}{r} 50 \\ \times 0 \\ \hline \end{array}$	$\begin{array}{r} 47 \\ \times 1 \\ \hline \end{array}$

Name: _____

More Practice With 0 and 1

The product of any number and zero is 0.



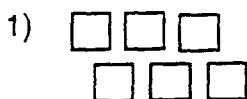
3 groups of 0
 $3 \times 0 = 0$

The product of any number and 1 is the number.

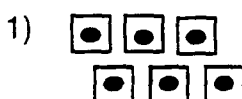


3 groups of 1
 $3 \times 1 = 3$

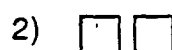
Complete each group



6 groups of 0
 $6 \times 0 = \underline{\hspace{2cm}}$



6 groups of 1
 $6 \times 1 = \underline{\hspace{2cm}}$



2 groups of 0
 $2 \times 0 = \underline{\hspace{2cm}}$



2 groups of 1
 $2 \times 1 = \underline{\hspace{2cm}}$

Now, find the product:



1)
$$\begin{array}{r} 0 \\ \times 7 \\ \hline \end{array}$$

2)
$$\begin{array}{r} 2 \\ \times 1 \\ \hline \end{array}$$

3)
$$\begin{array}{r} 8 \\ \times 1 \\ \hline \end{array}$$

4)
$$\begin{array}{r} 0 \\ \times 5 \\ \hline \end{array}$$

5)
$$\begin{array}{r} 5 \\ \times 1 \\ \hline \end{array}$$

6)
$$\begin{array}{r} 8 \\ \times 0 \\ \hline \end{array}$$

7)
$$\begin{array}{r} 1 \\ \times 15 \\ \hline \end{array}$$

8)
$$\begin{array}{r} 0 \\ \times 10 \\ \hline \end{array}$$

9)
$$\begin{array}{r} 1 \\ \times 1 \\ \hline \end{array}$$

10)
$$\begin{array}{r} 4 \\ \times 1 \\ \hline \end{array}$$

11) $0 \times 9 = \underline{\hspace{2cm}}$

12) $1 \times 12 = \underline{\hspace{2cm}}$

13) $6 \times 0 = \underline{\hspace{2cm}}$

14) $15 \times 1 = \underline{\hspace{2cm}}$

15) $0 \times 20 = \underline{\hspace{2cm}}$

16) $1 \times 9 = \underline{\hspace{2cm}}$

Name: _____

Multiplication Doubles

When you multiply by 2, think
addition doubles or **doubling**.

$$2 \times 6 = 12$$

THINK

$$6 + 6 = 12$$



$$6 + 6 = 12$$

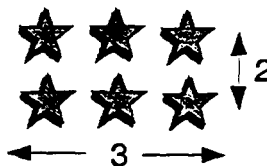
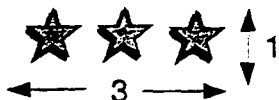
SO



$$2 \times 6 = 12$$

Double the array. Write an addition and a multiplication sentence.

Example



$$3 + 3 = 6$$

$$2 \times 3 = 6$$

1)



$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

2)



$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

3)



$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

4)



$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

Multiplication Doubles

Find the sum. Then, find the product.

$$\begin{array}{r} 1) \quad 6 \\ + 6 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ \times 6 \\ \hline \end{array}$$

$$\begin{array}{r} 2) \quad 4 \\ + 4 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ \times 4 \\ \hline \end{array}$$

$$\begin{array}{r} 3) \quad 9 \\ + 9 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 4) \quad 8 \\ + 8 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ \times 8 \\ \hline \end{array}$$

$$\begin{array}{r} 5) \quad 3 \\ + 3 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{r} 6) \quad 7 \\ + 7 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ \times 7 \\ \hline \end{array}$$

Write a multiplication sentence and solve:

$$\begin{array}{l} 1 + 1 \\ 2 \times 1 = 2 \end{array}$$

$$\begin{array}{l} 5 + 5 \\ ___ \times ___ = ___ \end{array}$$

$$\begin{array}{l} 2 + 2 \\ ___ \times ___ = ___ \end{array}$$

$$\begin{array}{l} 10 + 10 \\ ___ \times ___ = ___ \end{array}$$

Matching:

$$___ 2 \times 8 = \boxed{16}$$

$$___ 4 \times 2 = \square$$

$$___ 7 \times 2 = \square$$

$$___ 2 \times 2 = \square$$

$$___ 2 \times 9 = \square$$

$$___ 5 \times 2 = \square$$

$$___ 2 \times 1 = \square$$

$$___ 3 \times 2 = \square$$

$$___ 6 \times 2 = \square$$

a) $1 + 1$

b) $9 + 9$

c) $3 + 3$

d) $8 + 8$

e) $6 + 6$

f) $5 + 5$

g) $7 + 7$

h) $2 + 2$

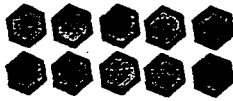
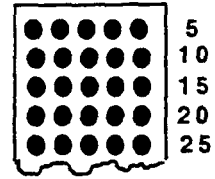
i) $4 + 4$

Remember:
When doubling in
multiplication, 2 is
always a **factor**.
 $2 \times 8 = 16$



Name: _____

Multiplication With Fives



2 groups

5 in each group

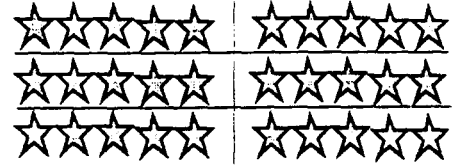
$$2 \times 5 = \square$$



3 groups

\square in each group

$$3 \times \square = \square$$

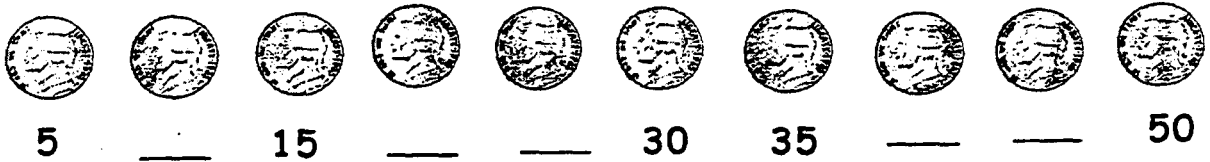


\square groups

\square in each group

$$\square \times \square = \square$$

Use nickels to count by fives.



Write a multiplication sentence.

How much money is 3 nickels? 15 ¢ $3 \times 5 = 15$

How much money is 4 nickels? _____ ¢ _____ \times _____ = _____

How much money is 6 nickels? _____ ¢ _____ \times _____ = _____

How much money is 9 nickels? _____ ¢ _____ \times _____ = _____

How much money is 2 nickels? _____ ¢ _____ \times _____ = _____

Find the product:

$$\begin{array}{r} 1) \quad 5 \\ \times 1 \\ \hline \end{array}$$

$$\begin{array}{r} 2) \quad 5 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 3) \quad 7 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 4) \quad 5 \\ \times 8 \\ \hline \end{array}$$

$$\begin{array}{r} 5) \quad 2 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 6) \quad 5 \\ \times 9 \\ \hline \end{array}$$

Objective 4: Multiply three single digit factors. Recognize and use the associative and distributive properties.

Vocabulary

associate
associative property
distribute
distributive property
factor
product
parentheses ()

Materials

chart paper
markers
counters

Transparencies:

The Distributive Property of Multiplication
Spring Concert
Multiplication Properties

Student Copies:

Three Factors and the Associative Property
Practice With the Distributive Property of
Multiplication
Practice With Multiplication Properties

Language Foundation

1. Review the verb to **associate**. Remind students that they associate with students in classes. Use your class as an example. There are (# of students) in this class. Sometimes students sit in rows, or sometimes they may sit in groups. Either way, the number of students remains the same even if the way they are grouped changes.

The **associative property** allows for different groupings, but the product or sum remains the same.

2. In this lesson, the verb to **distribute** means to spread out. Think of M & Ms in a bag. Suppose you want to eat all of the blue ones first. How will you get them all out? They will be easier to find if you empty the bag and spread them out.

The **distributive property** makes it easier to work with numbers if they are spread out.

Mathematics Component

Associative Property of Multiplication

1. Write the following on the board: $6 \times 3 = ?$

- Ask for a volunteer to come up and show how to find the answer to this problem. Tell them they can do anything they want to - draw a picture, make an array, add some numbers...
- Allow time for students to illustrate several different ways to show the product on the board.

Examples might include:

- a 6 by 3 array,
- a 3 by 6 array (because the Commutative Property of Multiplication says that the product would be the same if you change the order),
- 6 equal groups of 3 items,
- 3 equal groups of 6 items,
- $3 + 3 + 3 + 3 + 3 + 3$, or
- $6 + 6 + 6$.
- Write the following problem on the board: $2 \times 5 \times 5 = ?$
- Point to the first problem and ask students, "What is different about this problem?" (This problem has three **factors** instead of two.)
- Ask students what they think this problem means. Lead students to understand that the product of two of the factors must be multiplied by the third factor to get the final product.
- Put students into groups of 3 or 4 students. Give each group a large piece of chart paper and a marker.
- Tell students that you want them to work with their group to find the **product** of the three factors using any method that they can.
- Have groups find the product and illustrate how they got their answer on the chart paper provided.
- Allow time for each group to share their work.
- Most groups will work the problem in the order shown: $2 \times 5 \times 5 = 10 \times 5 = 50$. Lead students to understand that the two 5s may also be multiplied first to get 25. What remains is 25×2 . Although students have not learned double digit multiplication, they can think of the 25 as a quarter. To find the final product, they can find the value of 2 quarters. ($25¢ + 25¢ = 50¢$) The product of 25×2 is 50.
- Tell students that the **Associative Property of Multiplication** says that changing the grouping of the factors does not change the product.
- Go back to the original problem on the board.
- Tell students that we use special symbols in math to show which numbers are grouped together.
- Go back to the original problem on the board and show the following.

$$(2 \times 5) \times 5 = \underline{\text{and}}$$

$$2 \times (5 \times 5) =$$

(Note: Be sure to point out that the order has not changed, just the grouping is different.)

- Point and say, "These special grouping symbols are called **parentheses**. Parentheses group the numbers that will be done first."
- Walk students through the process of multiplying the numbers in the parentheses first.

$$\begin{array}{ccc} 1) & (2 \times 5) \times 5 = & \underline{\text{and}} \\ & 10 \times 5 = 50 & \end{array} \quad \begin{array}{ccc} 2) & 2 \times (5 \times 5) = & \\ & 2 \times 25 = 50 & \end{array}$$

- Discuss which groups used method 1 to find the product of $2 \times 5 \times 5$ and which students used method 2.
- Write $4 \times 5 \times 2$ on the board. Ask students to show two different ways that the numbers could be grouped to find the product. $4 \times (5 \times 2) = ?$ or $(4 \times 5) \times 2 = ?$
- Work through both problems and then have students vote on which method they prefer.

$$\begin{array}{ccc} 1) & (4 \times 5) \times 2 = & \underline{\text{and}} \\ & 20 \times 2 = 40 & \end{array} \quad \begin{array}{ccc} 2) & 4 \times (5 \times 2) = & \\ & 4 \times 10 = 40 & \end{array}$$

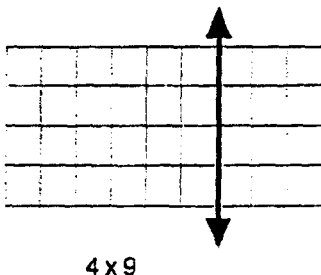
- The activity sheets Three Factors and the Associative Property are provided for additional practice.

Distributive Property of Multiplication

- Show students a handful of counters.
- Write the word distribute on the board and tell students that you will **distribute** the counters, or spread them out.
- Walk around the room and give one counter to each of several students. Say, "I have distributed the counters."
- Write the following on the board.

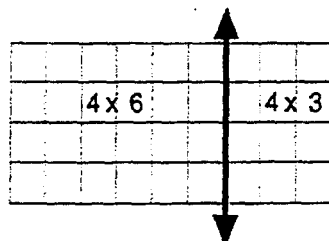
Distributive Property of Multiplication

- Explain that the Distributive Property of Multiplication helps us spread out numbers when we are multiplying. Underline the word *Distributive* and explain that it comes from the word distribute which means to spread out.
- Place the transparency The Distributive Property of Multiplication on the overhead.
- Have students name the array at the top of the page by counting its rows and columns. (4×9) Write the name below the array.
- Draw a vertical line through the array as shown below using a colored overhead marker.



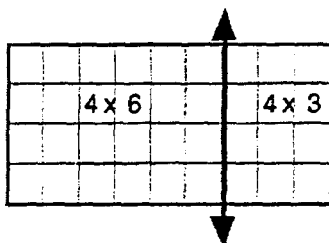
- Point and tell students that you have divided the array into two arrays.

Cover one part and have students name each array. Record as shown.



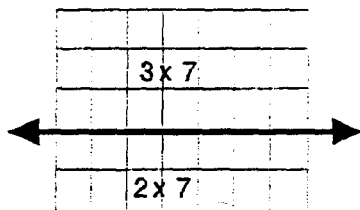
$$4 \times 9 =$$

- As you rewrite the 4×9 array as shown below say, "The 4×9 array has been distributed or spread out into two arrays - a 4×6 plus a 4×3 ."



$$4 \times 9 = (4 \times 6) + (4 \times 3)$$

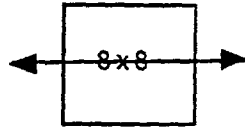
- Count the total number of squares in the 4×9 and show that there are the same number of squares if you add the 4×6 and 4×3 together.
- Repeat this same procedure with the second array, dividing it horizontally as shown below. Point out that the total array is equal to the **sum** of the two parts.



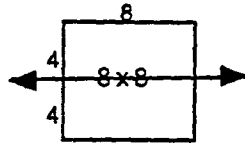
$$5 \times 7 = (2 \times 7) + (3 \times 7)$$

- The concept involved in the distributive property can be useful to students in figuring out basic facts.
 - Spend a few minutes discussing concerts which students have performed in. They may have been singing or playing an instrument in the band or orchestra.
 - Place the transparency, Spring Concert, on the overhead.
 - Explain that there is an 8×8 array of students sitting on the stage ready to perform in the spring concert. We want to know the number of students performing.
 - Tell students that the Distributive Property of Multiplication can help us find the total number of students in the array without counting them individually.
 - Say, "There are two ways the distributive property can help us. The first way is that we can cut the array into two parts."

- Draw a horizontal line through the middle of the array.

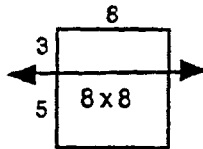


- Say, "The 8 x 8 array has been spread out into two arrays. What would the two arrays be called?" (They would be a 4 x 8 and a 4 x 8 if they were divided horizontally in the middle.)
- Fill in the appropriate number sentence as shown below.



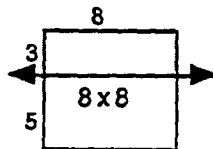
1) <u>Cut the array into two parts.</u> $8 \times 8 = (4 \times 8) + (4 \times 8)$	2) <u>Use a fact you know.</u>
---	--------------------------------

- Erase the horizontal line and go on to the second way the Distributive Property can help us find a product. Lead students to understand that since 5 facts are easy to remember, we can start with 5 x 8.
- Discuss where the line would go to divide the 8 x 8 array into a 5 x 8.
- Draw the line and then ask students to name the two arrays. (5 x 8 and 3 x 8) Record as shown below. Point out that 5 + 3 = 8. These two parts form the whole side, 8.



1) <u>Cut the array into two parts.</u> $8 \times 8 = (4 \times 8) + (4 \times 8)$	2) <u>Use a fact you know.</u> $8 \times 8 = (5 \times 8) + (3 \times 8)$
---	--

- Help students work each side to find the total number of students in the 8 x 8 array.



1) <u>Cut the array into two parts.</u> $8 \times 8 = (4 \times 8) + (4 \times 8)$ $8 \times 8 = 32 + 32$ $8 \times 8 = 64$	2) <u>Use a fact you know.</u> $8 \times 8 = (5 \times 8) + (3 \times 8)$ $8 \times 8 = 40 + 24$ $8 \times 8 = 64$
--	---

- Discuss other ways the 8 x 8 array could be spread out to find its product. $(7 \times 8) + (1 \times 8)$
 $(6 \times 8) + (2 \times 8)$

- Additional practice is provided on the activity sheet Practice With the Distributive Property of Multiplication. A review of all the properties is provided on the sheets Practice With Multiplication Properties. A transparency master/wall poster, Multiplication Properties, is also provided for review.

Name: _____

Three Factors and the Associative Property

You can multiply three factors in any order. The product will be the same.
 $(2 \times 2) \times 5 = 2 \times (2 \times 5)$

Find the product.

$(1 \times 2) \times 9 =$

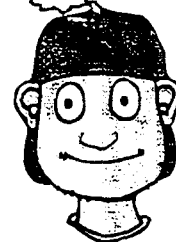
$(6 \times 0) \times 8 =$

$(5 \times 2) \times 3 =$

$1 \times (2 \times 9) =$

$6 \times (0 \times 8) =$

$5 \times (2 \times 3) =$



Group two factors with parentheses () then multiply.

1) $4 \times 2 \times 4 =$ _____

5) $2 \times 6 \times 2 =$ _____

2) $1 \times 2 \times 3 =$ _____

6) $8 \times 0 \times 9 =$ _____

3) $5 \times 5 \times 1 =$ _____

7) $4 \times 4 \times 2 =$ _____

4) $9 \times 3 \times 2 =$ _____

8) $3 \times 7 \times 1 =$ _____

Rewrite a multiplication sentence using (). Then solve.

Example: $(4 \times 1) \times 3 = \underline{12}$

$4 \times (1 \times 3) = 12$

1) $(6 \times 2) \times 3 =$ _____

2) $(2 \times 5) \times 4 =$ _____

3) $(1 \times 2) \times 7 =$ _____

4) $(6 \times 4) \times 2 =$ _____

Now use numbers to write and solve your own multiplication sentences:

5) $(\text{ } \times \text{ }) \times \text{ } = \text{ }$ _____

6) What happens to the product when you change the ways the factors are grouped?

Three Factors and the Associative Property

Choose the missing factor from the numbers in the box.



1) $(1 \times 4) \times \underline{\hspace{1cm}} = 12$

4) $(2 \times 3) \times \underline{\hspace{1cm}} = 36$

2) $(5 \times \underline{\hspace{1cm}}) \times 2 = 50$

5) $\underline{\hspace{1cm}} \times (1 \times 3) = 24$

3) $(\underline{\hspace{1cm}} \times 4) \times 2 = 32$

6) $(1 \times 3) \times \underline{\hspace{1cm}} = 21$

Complete the problem.

1) $9 \times \underline{\hspace{1cm}} \times 4 = 0$

4) $\underline{\hspace{1cm}} \times 9 \times 4 = 36$

2) $5 \times 3 \times \underline{\hspace{1cm}} = 15$

5) $0 \times 5 \times 7 = \underline{\hspace{1cm}}$

3) $4 \times 2 \times 1 = \underline{\hspace{1cm}}$

6) $5 \times 5 \times \underline{\hspace{1cm}} = 25$

Problem Solving

Use the Associative Property of Multiplication to help you

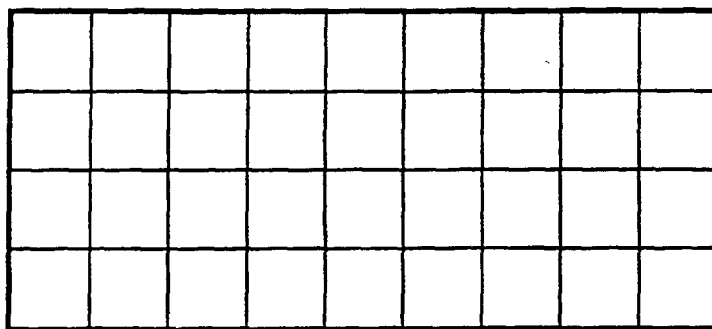
There are 5 boxes on the table. Each box has 2 cups in it. There are 5 M & M's in each cup. How many M & M's are there in all?
(Hint: draw a model.)



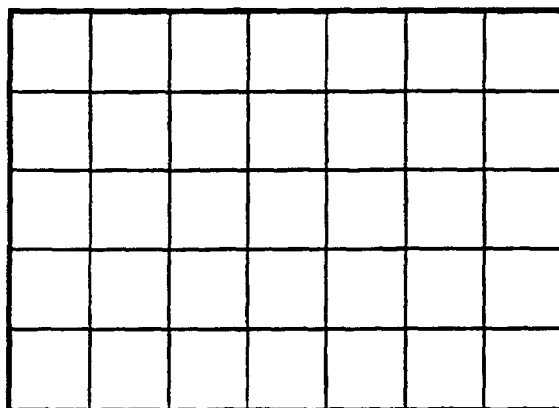
Marla is having a birthday party. 2 people are bringing birthday cakes. Those two people each bake 3 cakes! Each cake has 10 candles. How many candles are there on all the cakes? (Be sure to draw a model!)



The Distributive Property of Multiplication

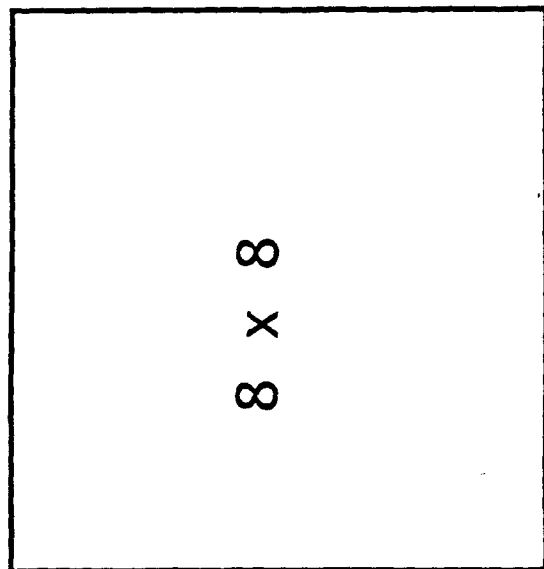
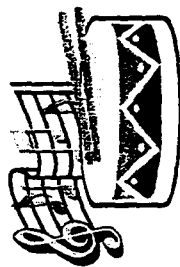
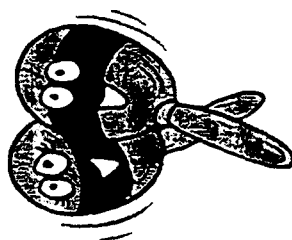
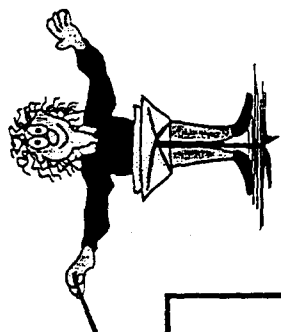


$$4 \times 9$$



$$5 \times 7$$

Spring Concert



$$8 \times 8$$

1) Cut the array into two parts

2) Use a Fact you know

Name: _____

Practice With the Distributive Property of Multiplication

$$\begin{array}{c}
 6 \times 7 \\
 \swarrow \quad \searrow \\
 (6 \times 2) + (6 \times 5)
 \end{array}$$

If you do not remember a fact, use the Distributive Property to rewrite the problem in an easier way.

The Distributive Property says you can break apart numbers to make them easy to multiply.

- Using the Distributive Property, cut the array into two smaller parts to help find the product.
- Write a multiplication sentence for each part, then add. There will be several ways to cut the array.



Examples:

6 x 4 = ?

6 x 2 = 12

6 x 2 = 12

Add 12 + 12 = 24

or

6 x 4 = ?

2 x 4 = 8

4 x 4 = 16

Add 8 + 16 = 24

1) 5 x 5

_____ x _____ = _____

_____ x _____ = _____

Add _____ + _____ = _____

5 x 5 = _____

2) 8 x 6

_____ x _____ = _____

_____ x _____ = _____

Add _____ + _____ = _____

8 x 6 = _____

3) 11 x 4

_____ x _____ = _____

_____ x _____ = _____

Add _____ + _____ = _____

11 x 4 = _____

4) 5 x 7

_____ x _____ = _____

_____ x _____ = _____

Add _____ + _____ = _____

5 x 7 = _____

Name: _____

Practice With Multiplication Properties



Distributive Property

Associative Property

Commutative Property

Zero Property

Identity Property

Write the name of the property used in each number sentence.

- 1) $7 \times 4 = 4 \times 7$ _____
- 2) $32 \times 1 = 32$ _____
- 3) $(8 \times 5) \times 6 = 8 \times (5 \times 6)$ _____
- 4) $77 \times 0 = 0$ _____
- 5) $3 \times 15 = (3 \times 10) + (3 \times 5)$ _____

Complete and write the name of the property.

- 1) _____ $\times 435 = 0$ _____
- 2) $4 \times (2 \times 1) = (4 \times 2) \times \text{_____} = 8$ _____
- 3) $3 \times 9 = (3 \times 3) + (3 \times \text{_____})$ _____
- 4) $96 \times \text{_____} = 96$ _____
- 5) $15 \times 7 = \text{_____} \times 15$ _____

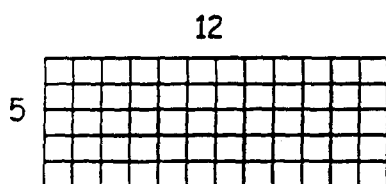
Practice With Multiplication Properties



Matching math language: Write the letter in the blank.

- | | |
|---|---|
| 1) The answer to a multiplication problem. | A) Distributive Property |
| 2) Equal rows and equal columns. | B) Identity Property ($8 \times 1 = 8$) |
| 3) If the order of the factors is changed, the product stays the same. | C) Product |
| 4) The numbers we multiply to get a product. | D) The other factor is two. |
| 5) You can break numbers apart to make multiplication easier. | E) Zero Property ($8 \times 0 = 0$) |
| 6) The factors are six and two. | F) Factors |
| 7) If one factor is zero, the product is zero. | G) Associative Property |
| 8) If the grouping of the factors is changed, the product stays the same. | H) Commutative Property |
| 9) The product is ten. One factor is five. | I) The product is twelve |
| 10) If you multiply by one, the product is always the same as the other factor. | J) Array |

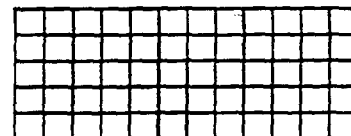
Lets Talk About the Distributive Property



- 1) Write a multiplication sentence for this array.

- 2) How many units are inside this array? _____

- 3) Now, count off 10 units across the top of the array.
Draw a vertical line to cut apart the rectangle.



4) How many units are in each of the two arrays?

Array 1 _____

Array 2 _____

5) Write a multiplication sentence for each array.

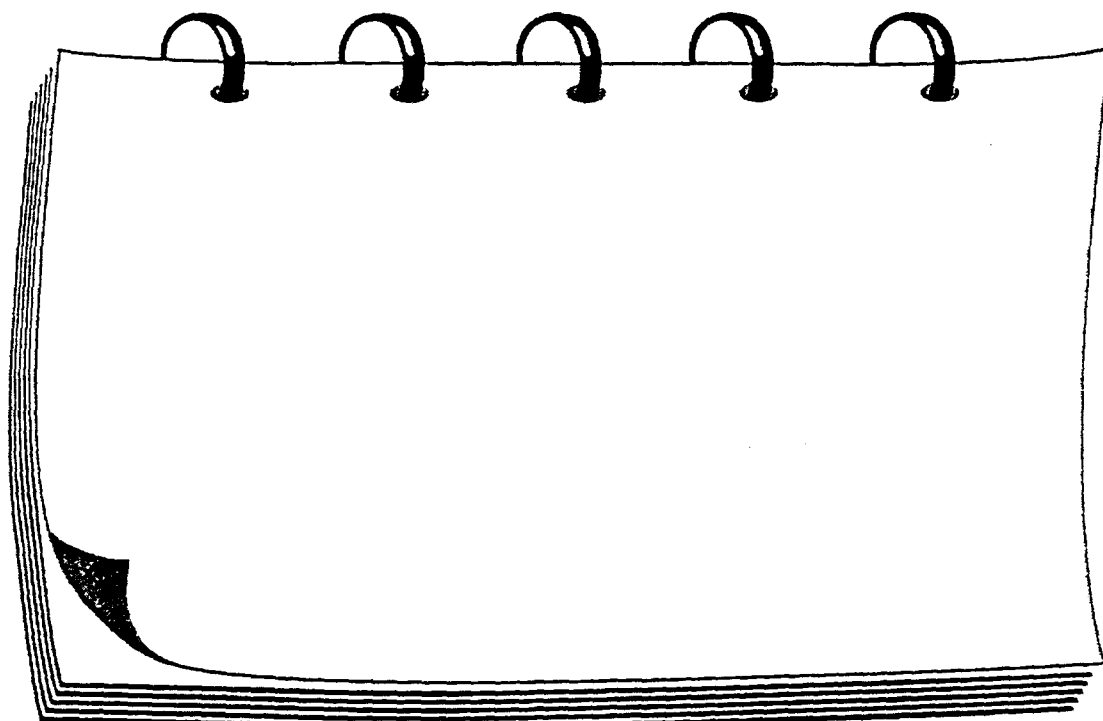
Array 1 _____

Array 2 _____

6) What is the sum of the products of the two smaller arrays? _____

7) Is the product of the large array the same as the sum of the products for the two small arrays? _____

8) Explain to a friend how you can use the Distributive Property of Multiplication to help you multiply when you have big numbers. Draw a model to help you show your friend what to do. Try to use math vocabulary words, too!



Multiplication Properties

Commutative Property of Multiplication

The order of the factors
does not matter.

$$6 \times 8 = 8 \times 6$$

Associative Property of Multiplication

You can group factors
differently. The product
is the same.

$$(3 \times 1) \times 2 = 3 \times (1 \times 2)$$

Distributive Property of Multiplication

You can break apart
numbers to make
multiplying easier.

$$4 \times 9 = (4 \times 3) + (4 \times 6)$$

Identity Property

If a factor is 1, the
product equals the other
factor.

$$5 \times 1 = 5$$

Zero Property

If one factor is 0, the
product is 0.

$$7 \times 0 = 0$$

Objective 5: Use models to multiply 2-digit numbers by 1-digit numbers.

Vocabulary

array
repeated addition
row
column
earn

Language Foundation

1. One of the word problems contains the verb **earn**. Many students will be familiar with this word, especially if they have a job. Explain that when you work, you get money in return for the work that you do. You earn that money.

Materials

base ten blocks
transparent base ten blocks
centimeter graph paper

Student Copies:

Exploring Multiplication With 2-Digit
Numbers
Model and Multiply
Multiplying 2-Digit Numbers

Mathematics Component

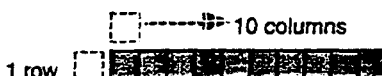
Note: There are several algorithms, or written procedures, for multiplication. Students often find these procedures abstract and difficult to understand. Many times they learn isolated rote procedures with no ability to transfer their understanding to other tasks. Working with concrete materials such as base-ten blocks focuses students on conceptual development and ensures more successful experiences with the traditional written procedures.

1. Distribute bags of base-ten blocks (2 tens and 25 ones) to each pair of students.

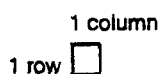
- Place a transparent ten, or rod, on the overhead. Tell students that ones have been put together to make an array.



- Ask students to name the number of rows and columns in this array. (1 x 10)

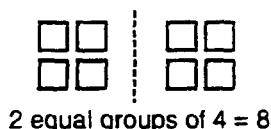


- Have students name the total number of squares in a 1 x 10. (10) Review that, "This is called a ten."
- Place a transparent one, or unit, on the overhead. Ask students to name the number of rows and columns in this array. (1 x 1)

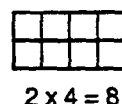


- Have students name the total number of squares in a 1 x 1. (1) Review that, "This is called a one."
- Write $2 \times 4 = ?$ on the board. Use overhead ones to review two methods of showing the product.

Repeated Addition



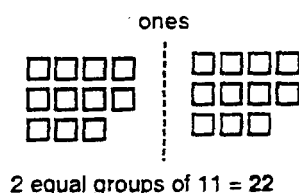
Array



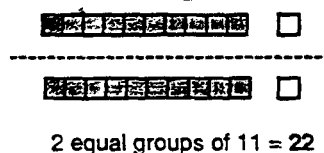
2. Write $2 \times 11 = ?$ on the board. Ask students how we can illustrate the product using base ten blocks. Lead them to understand that we can use repeated addition (combining equal groups) or an array.

- Have students work with a partner. Begin with repeated addition and say, "Can you use base-ten blocks to show me 2 equal groups of 11?"
- Model both ways that students might show 2 equal groups of 11.

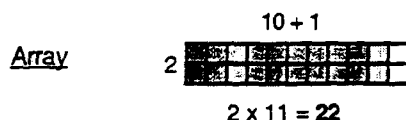
Repeated Addition



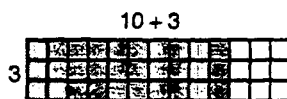
tens and ones



- Have students count the total number of blocks used to make 2 equal groups of 11. (22)
- Say, "Two times eleven equals twenty-two." Write $2 \times 11 = 22$.
- Point and ask students which they think is easier - counting out 11 ones each time or using tens and ones to make 11. (Students should agree that using tens and ones is a little faster.)
- Tell students that both are correct, but using tens and ones makes our work faster.
- Have students who used all ones replace theirs with tens and ones.
- Ask students if they can make a 2×11 array using the tens and ones. After students have had time to work with a partner, have one group model the solution.



- Ask students how we can change this to an 11×2 array. (Turn the array around so that it has 11 rows and 2 columns.)
- Say, "What is the product of 11×2 ? Why?" (The product is 22, because the Commutative Property of Multiplication says that if the order of the factors changes, the product stays the same.)
- Write 3×13 on the board.
- Ask students to make an array using tens and ones to find the product of 3×13 .
- Have a student model the solution on the overhead.

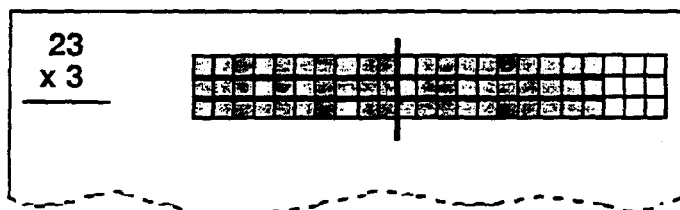


- Say, "Three times thirteen equals thirty-nine." Write $3 \times 13 = 39$. "What is 13×3 ?" (39)
- Write the following problems on the board and ask students to make arrays to find the products.

1) $4 \times 22 =$ 2) $2 \times 14 =$ 3) $31 \times 3 =$ 4) $44 \times 2 =$

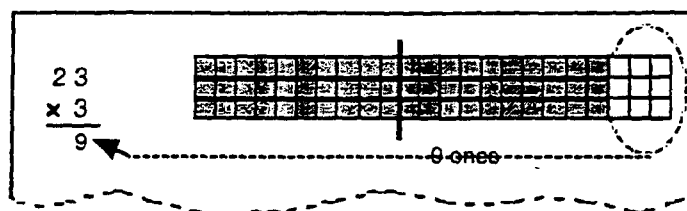
- Allow students to model correct solutions.

Place a clean transparency on the overhead. Write the following problem to the side of the transparency. Model a 23×3 array beside the problem.

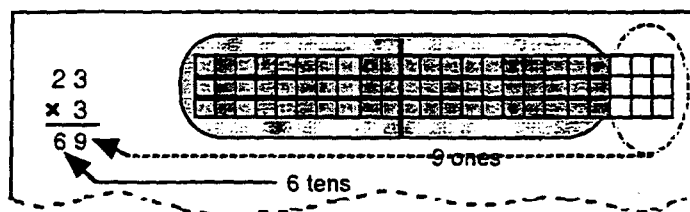


- Remind students that multiplication problems can be written in both a horizontal and a vertical way.
- Review that the number 23 has 2 tens and 3 ones. There are three rows with 23 in each row.

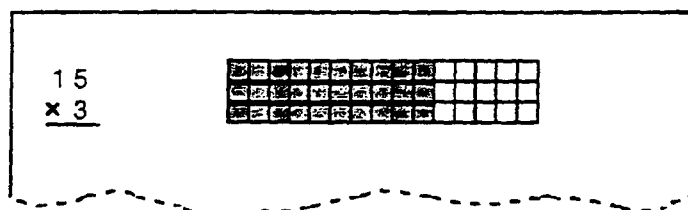
- Tell students that we begin with the ones to record the product. Say, "There are three ones in each row and there are three rows. How many ones are in a 23×3 array?" (9) Circle the ones in the array with a colored pen and record the number of ones in the product.



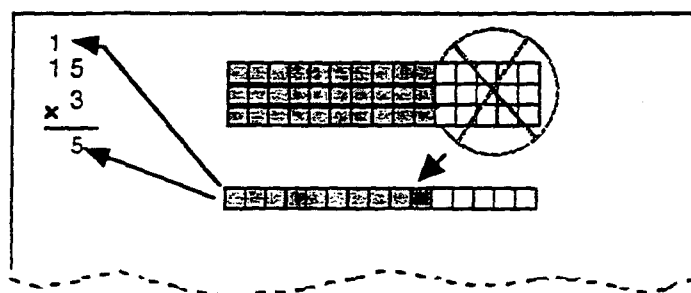
- Repeat the same procedure for the tens. Say, "There are two tens in each row and there are three rows. How many tens are in a 23×3 array?" (6) Circle the tens in the array with a different colored pen and record the number of tens in the product.



- Remove the problem and the array on the transparency. Set up the following problem.



- Remind students that we begin with the ones to record the product. Say, "There are five ones in each row and there are six rows. How many ones are in a 15×3 array?" (15) Circle the ones in the array.
- Regroup and show students that 15 ones are the same as 1 ten and 5 ones. Explain that we will record the 5 ones under the ones column in the product and put the 1 ten above the tens column.



- To record the number of tens, circle the tens and say, "There are three rows with one ten in each row **plus** one more ten which was regrouped from the ones. How many tens are there altogether?" (4) Record 4 tens in the product.
- Distribute a piece of centimeter graph paper to each student. Write the following problems on the board:

$$1) 27 \times 2 \quad 2) 16 \times 3 \quad 3) 2 \times 46 \quad 4) 4 \times 18$$

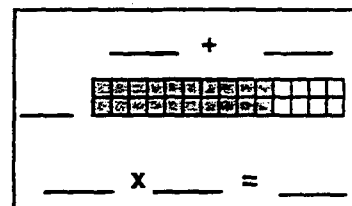
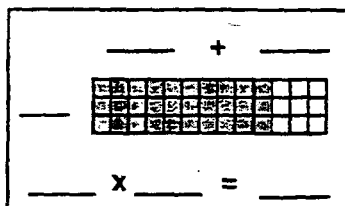
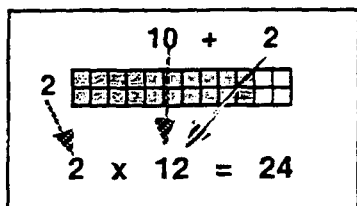
- Have students sketch arrays on the graph paper to find and record the product for each problem. Remind students to watch for places to regroup!
- The activity pages Exploring Multiplication With 2-Digit Numbers, Model and Multiply, and Multiplying 2-Digit Numbers will provide further practice with forming arrays to find products.

Note: Some students may need additional practice before they are able to apply the algorithm without concrete materials.

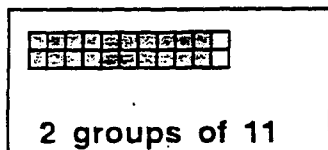
Name: _____

Exploring Multiplication With 2-Digit Numbers

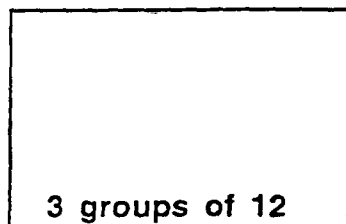
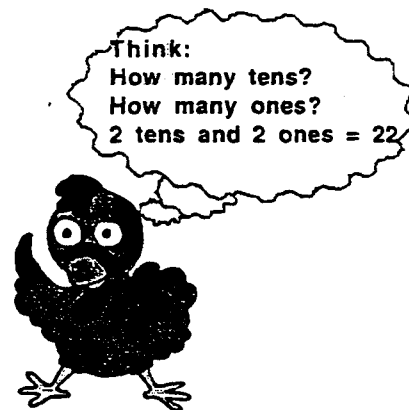
Use the models to label the array. Count the tens and ones.
Write a multiplication sentence and solve.



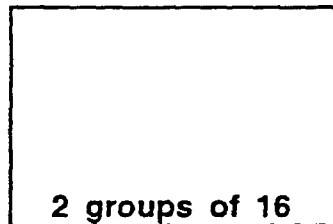
Draw the models. Count the ones and tens.
(Watch for regrouping !!!)
Write a multiplication sentence and solve.



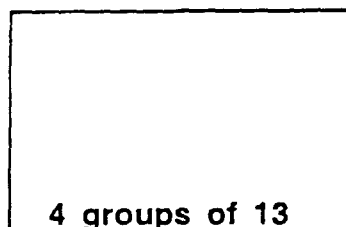
$$\begin{array}{r} 11 \\ \times 2 \\ \hline 22 \end{array}$$



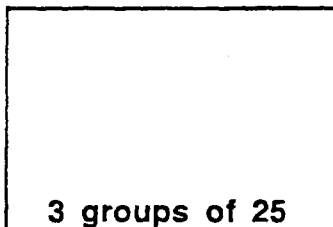
$$\begin{array}{r} ____ \\ \times ____ \\ \hline \end{array}$$



$$\begin{array}{r} ____ \\ \times ____ \\ \hline \end{array}$$



$$\begin{array}{r} ____ \\ \times ____ \\ \hline \end{array}$$



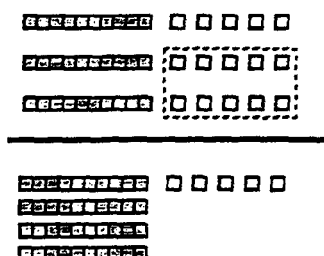
$$\begin{array}{r} ____ \\ \times ____ \\ \hline \end{array}$$

Name: _____

Model and Multiply

Find the product of 3×15

TENS	ONES
<div style="border: 1px solid black; padding: 2px;">1</div>	
1	5
x	3
4	5



- Multiply the ones $\rightarrow 3 \times 5 \text{ ones} = 15$
- Regroup 15 ones $\rightarrow 1 \text{ ten } 5 \text{ ones}$
- Multiply the tens $\rightarrow 3 \times 1 \text{ ten} = 3 \text{ tens}$
- Add the regrouped ten $\rightarrow 3 + 1 = 4 \text{ tens}$

Find the product. Use cubes and rods to make a model.

1)

TENS	ONES
<div style="border: 1px solid black; padding: 2px;"> </div>	
1	7
x	5

2)

TENS	ONES
<div style="border: 1px solid black; padding: 2px;"> </div>	
3	2
x	3

3)

TENS	ONES
<div style="border: 1px solid black; padding: 2px;"> </div>	
1	9
x	2

4)

TENS	ONES
<div style="border: 1px solid black; padding: 2px;"> </div>	
1	2
x	7

5)

TENS	ONES
<div style="border: 1px solid black; padding: 2px;"> </div>	
3	8
x	2

6)

TENS	ONES
<div style="border: 1px solid black; padding: 2px;"> </div>	
2	9
x	3

7)

TENS	ONES
<div style="border: 1px solid black; padding: 2px;"> </div>	
4	6
x	2

8)

TENS	ONES
<div style="border: 1px solid black; padding: 2px;"> </div>	
2	3
x	4

9)

TENS	ONES
<div style="border: 1px solid black; padding: 2px;"> </div>	
1	5
x	6